PRABHAT KUMAR COLLEGE, CONTAI

M. Sc. 1st Semester Examination-2021

Subject: Physics Paper: **PHS 101** Full Marks: 50 Time: 2 hr

101.1: Methods of Mathematical Physics-I

Answer any TWO questions

1. a) Establish Cauchy- Riemann (C-R) equations in polar form and use it to prove that the function $f(z) = Arg\, z,\ z \epsilon\ \mathbb{C}\text{-}\{\Theta\}$ is nowhere analytic, where *Arg z* denotes the principal value of argument with *z*.

 b) Prove that the function $f(z) = \sqrt{|Imz.Rez|}$, where $z\ \epsilon\ \mathbb{C}$ satisfies C-R equations at the origin, but it is not differentiable at origin. [5+5]

2. a) Evaluate $\int_0^\infty \frac{\sin x}{x} dx$

 b) Evaluate $\int \frac{dz}{(z^2+4)^2}$ over *c*, where *c: |z-i|=2*

 c) Find the Laurent series expansion of the function $f(z) = \frac{1}{(z-1)(4z^2+1)}$, in the annulus $\frac{1}{2} < z < 1$. [4+3+3]

3. a) State and prove Schwarz's inequality in a complex inner product space.

 b) Prove that the set of vectors {(1, 2, 2), (2, -2, 1), (2, 1, -2)}is an orthogonal basis of the Euclidean space R^3 with standard inner product. Use it to express the vector (4, 3, 2) as a linear combination of these basis vectors.

 c) State Cayley-Hamilton's theorem and use it to find A^9, where

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

[4+3+3]

4. a) Solve in series the Bessel differential equation:

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$$

Where *n* is not an integer.

 b) Find the value of $\Gamma(-3/2)\Gamma(1/2)$.

c) Prove that $\int_0^\infty e^{-x^2-2bx}\, dx = \frac{\sqrt{\pi}}{2} e^{b^2}. erf_c(b)$, where $erf_c(x)$ denotes the complementary error function of the error function *erf(x)*. [4+3+3]

Internal Assesment-05

101.2: Classical Mechanics

Answer any TWO questions

1. (a) What is cyclic coordinate? Discuss with an example. (b) Discuss the conservation of generalised momentum in this context and explain how conservation of linear momentum leads to homogeneity of space. 3+2+5

2. (a) Write down the Lagrangian of a spring-mass system and deduce the equation of motion. (b) Hence deduce the Hamiltonian for the system. (c) Discuss the form of the equation if the spring constant becomes a linear function of the expansion (x). 3+4+3

3. Consider two identical masses m connected to two fixed walls by two identical massless springs of spring constant k and coupled to each other by another massless spring of spring constant k'. Write down the Lagrangian of the system and solve to obtain the oscillation frequencies. Also find the normal coordinates and normal frequencies for the system. 10

4. Deduce the Lagrangian for a charge particle moving in an electromagnetic field. 10

Internal Assesment-05