

9. Answer any one question : 1×10

(a) (i) If  $\vec{r} = \vec{a} \cos nt + \vec{b} \sin nt$ , where  $\vec{a}$ ,  $\vec{b}$ ,  $n$  are constants, then prove that  $\frac{d^2 \vec{r}}{dt^2} + n^2 \vec{r} = \vec{0}$  and

$$\vec{r} \times \frac{d\vec{r}}{dt} = n \left( \vec{a} \times \vec{b} \right). \quad 5$$

(ii) Derive the volume of a tetrahedron whose coordinates of vertices are given. Use it to calculate the volume of the tetrahedron whose vertices are A(2, -1, -3), B(4, 1, 3), C(3, 3, -1) and D(1, 4, 2). 5

(b) (i) Prove that  $\left[ \left( \vec{\alpha} \times \vec{\beta} \right) \cdot \left( \vec{\beta} \times \vec{\gamma} \right) \cdot \left( \vec{\gamma} \times \vec{\alpha} \right) \right] = \left[ \vec{\alpha} \cdot \vec{\beta} \cdot \vec{\gamma} \right]^2$ ,

where  $[ \cdot ]$  denotes the scalar triple product. 5

(ii) Find  $\hat{t}$ ,  $\hat{n}$  for the curve given by

$$\vec{r} = (e^t \cos t, e^t \sin t, e^t) \text{ at } t = 0. \quad 5$$

2018

2nd Semester

MATHEMATICS

PAPER—C4T

(Honours)

Full Marks : 60

Time : 3 Hours

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

**Differential Equations and Vector Calculus**

**Unit-I**

[Marks : 22]

1. Answer any one question : 1×2

(a) Let  $\phi$  be a solution for  $0 < x < a$  of the Euler equation  $x^2 y'' + axy' + by = 0$  where  $a, b$  are constants. Let  $\psi(t) = \phi(e^t)$ , then show that  $\psi$  satisfies the equation

$$\frac{d^2 \psi}{dt^2} + (a-1) \frac{d\psi}{dt} + b\psi = 0.$$



- (b) Test whether the solution  $e^x$ ,  $e^{2x}$ ,  $e^{3x}$  are linearly independent or not.

2. Answer any two questions : 2×5

- (a) Knowing that  $y = x$  is a solution of the equation

$$x^2 \frac{d^2y}{dx^2} - x(x+2) \frac{dy}{dx} + (x+2)y = 0 \quad (x \neq 0)$$

reduce the equation

$$x^2 \frac{d^2y}{dx^2} - x(x+2) \frac{dy}{dx} + (x+2)y = x^3 \quad (x \neq 0)$$

to a differential equation of first order and first degree and find its complete primitive.

- (b) Solve the differential equation :

$$\frac{d^3y}{dx^3} + 2 \frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \cos x.$$

- (c) Solve the equation

$$\frac{d^2y}{dx^2} + a^2y = \tan ax$$

by the method of variation of parameters.

3. Answer any one question : 10×1

- (a) (i) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = x^2 \sin 2x$$

by the method of undetermined co-efficients. 5

- (ii) State the sufficient condition for existence and uniqueness of the solution of the differential

$$\text{equation } \frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0.$$

Show that  $\frac{dy}{dx} = \frac{1}{y}$ ,  $y(0) = 0$  has more than one solution and indicate the possible reason.

2+2+1

- (b) (i) Let  $a_1$ ,  $a_2$  are continuous functions on  $[a, b]$  and  $\phi_1$ ,  $\phi_2$  be the two independent solutions of  $y''(x) + a_1(x)y'(x) + a_2(x)y(x) = 0$  on some interval  $[a, b]$ . Let  $x_0$  be any point in  $[a, b]$ . Then show that

$$W(\phi_1, \phi_2)(x) = \exp \left\{ - \int_{x_0}^x a_1(t) dt \right\} W(\phi_1, \phi_2)(x_0),$$

$$\forall x \in [a, b]$$

$$\text{where } W(\phi_1, \phi_2)(x) = \begin{vmatrix} \phi_1(x) & \phi_2(x) \\ \phi_1'(x) & \phi_2'(x) \end{vmatrix} \quad 5$$



(ii) Solve the differential equation

$$x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2} \quad 5$$

### Unit-II

[Marks : 13]

4. Answer any four questions : 2×4

(a) Solve the equations  $\frac{dx}{dt} = -wy$  and  $\frac{dy}{dt} = wx$  and show that the point  $(x, y)$  lies on a circle.

(b) Solve the equation

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

(c) Find the complementary function for the system

$$(D+3)x + Dy = \cos t$$

$$(D-1)x + y = \sin t$$

where  $D \equiv \frac{d}{dt}$ .

(d) Solve :  $\frac{yzdx}{y-z} = \frac{zxdy}{z-x} = \frac{xydz}{x-y}$

(e) Show that the solution of the differential equations

$$\frac{dx}{dt} = 2x + y \quad \text{and} \quad \frac{dy}{dt} = 3x \quad \text{satisfies the relation}$$

$3x + y = ke^{3t}$  where  $k$  is a real constant.

(f) If  $\frac{dy_1}{dx} = 3y_1 + 4y_2$  and  $\frac{dy_2}{dt} = 4y_1 + 3y_2$  then find the value of  $y_1(x)$ .

5. Answer any one question : 5×1

(a) Find the fundamental matrix and the complementary solution of the homogenous linear system of differential equations

$$\frac{dx}{dt} = 3x + y \quad \text{and} \quad \frac{dy}{dt} = x + 3y \quad 5$$

(b) (i) Solve the equation

$$(x^2 + y^2 + z^2)dx - 2xydy - 2xzdz = 0.$$

(ii) Find  $f(y)$  such that the total differential

$$\frac{yz+z}{x} dx - zdy + f(y)dz = 0$$

is integrable. Hence solve it.  $2\frac{1}{2} + 2\frac{1}{2}$



## Unit-III

[Marks : 9]

6. Answer any *two* questions : 2×2

(a) Consider the set of non-linear differential equations

$$\frac{dx}{dt} = x - xy; \quad \frac{dy}{dt} = -y + xy.$$

Find the equilibrium points of the system of equations.

(b) Show that  $x = 0$  is a ordinary point and  $x = 1$  is a regular singular point of the ODE

$$x(x-1)\frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} + 2x(x-1)y = 0.$$

(c) What do you mean by stable and constable critical points.

7. Answer any *one* question : 5×1(a) Find the phase curve of the system of dynamical equations  $\dot{x} = -x - 2y$  and  $\dot{y} = 2x - y$ . Also show that the system is stable.

(b) Find the power series solution of the equation

$$(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0$$

in power of  $x$  about the origin.

## Unit-IV

[Marks : 16]

8. Answer any *three* questions : 3×2

(a) Show that the vector

$$\vec{F} = (2x - yz)\hat{i} + (2y - zx)\hat{j} + (2z - xy)\hat{k}$$

is irrotational.

(b) Test the continuity of the vector function

$$\vec{f}(t) = |t|\hat{i} - \sin t\hat{j} + (1 + \cos t)\hat{k} \quad \text{at } t = 0.$$

(c) If  $\vec{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20z^2x\hat{k}$ , evaluate  $\int_c \vec{A} \cdot d\vec{r}$  from(0, 0, 0) to (1, 1, 1) along the path  $c : x = t, y = t^2, z = t^3$ .

(d) Find the unit vector in the direction of the tangent at any point on the curve given by

$$\vec{r} = (a \cos t)\hat{i} + (a \sin t)\hat{j} + bt\hat{k}.$$

(e) Show that the vectors  $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$  are coplanar.