Advacned Management Accounting
For M.Com Semester –III
Capital Budgeting

By
Tamal Basu
Assistant Professor
Department of Commerce (UG & PG)
Prabaht Kumar College, Contai

Introduction

- Capital Budgeting, broadly defined as a decision-making process that enables managers to evaluate and recognize projects that are valuable to the company, is usually the dominant mission facing any financial manager and his/her team.

What is Capital Budgeting?

- Capital Budgeting is the process by which the firm decides which long-term investments to make. Capital Budgeting projects, i.e., potential long-term investments, are expected to generate cash flows over several years.

The Basic Steps of Capital Budgeting

- **Capital Budgeting Basics:** A company undertakes capital budgeting in order to make the best decisions about utilizing its limited capital.
- **Identify Potential Opportunities:** The first step in the capital budgeting process is to identify the opportunities that you have.
- **Evaluate Opportunities:** Once you have identified the reasonable opportunities, you need to determine which ones are the best.
- **Cash Flow:** Next, you need to determine how much cash flow it would take to implement a given project.

Needs

1. **Long-term Implication:** Capital expenditure decision affects the company's future cost structure over a long time span.
2. **Irreversible Decision:** Capital investment decision are not easily reversible without much financial loss to the firm because there may be no market for second-hand plant and equipment and their conversion to other uses may not be financially viable.
3. **Long-term Commitments Of Funds:** Capital budgeting decision involves the funds for the long-term. So, it is long-term investment decision.

Types of Capital budgeting

1) **Accept reject decisions:** all the investment decisions which give more return than the cost of capital they are acceptable while the investment decisions which give less return than the cost of capital they are rejected.
2) **Mutually exclusive decisions:** these are the decisions which compete with each other which mean the acceptance of one automatically rejects the other decision..
3) **Capital rationing or ranking decisions:** in case the firm has various profitable investment proposals in that case the firm had only option to rank them as per their profitability and then accept them.

Nature of Capital Budgeting

(a) Capital expenditure plans involve a huge investment in fixed assets.

(b) Capital expenditure once approved represents long-term investment that cannot be reserved or withdrawn without sustaining a loss.

(c) Preparation of coital budget plans involve forecasting of several years profits in advance in order to judge the profitability of projects.

Benefits of Capital Budgeting

1. Consistency and flexibility
2. Better financial decisions
3. Access risk and uncertainty
4. Analyze long-term repercussions

Factors influencing Capital Budgeting

Capital Budgeting refers to the techniques used for making long-term investment decisions. Capital budgeting is defined "as the firm's formal process for the acquisition and investment of capital. It involves firm's decisions to invest its current funds for addition, disposition, modification and replacement of fixed assets".

The factors influencing capital budgeting are:

- Availability of funds
- Structure of capital
- Taxation policy
- Government Policy
- Lending policies of financial institutions
- Immediate need of the project
- Working capital
- Trend of earnings
- Capital return

Capital Budgeting Techniques

- Capital Budgeting Techniques include:

Payback Period (PB)
Internal Rate of Return (IRR)
Net Present Value (NPV)
Discounted Payback Period (DPB)
Profitability Index (PI)

Pay Back Period

- Pay Back Period is the necessary period for giving back the invested money. It emphasizes more on annual cash inflows, economic life of the project and original investment.

		Cash Flows	Cumulative Cash inflow
A	Cash outflow	(100,000)	(100,000)
B	Cash inflow		
	1st Year	45,000	45,000
	2nd Year	40,000	85,000
	3rd Year	35,000	120,000
	4th Year	50,000	170,000

Pay Back Period (PBP)

- PBP = Years + Remaining amount/Cash inflow in the next year
- PBP = 2 + (100,000 – 85,000) / 35,000 = 2.428
- So, above calculation reveals that, the calculated PBP is 2.428 (2 years 5 months and 5 days)

- Limitations of PBP

It does not consider the time value of money

It does not consider the cash flow after PBP

It does not consider profitability of economic life of project

It does not not recognize pattern of cash flow

It does not reflect all the relevant dimensions of profitability

Payback Period

Sometimes called the payout method i.e., a computationally simple project evaluation approach that has been used for many years. The procedure is to determine how long it takes a project to return the cost of the original investment.

Example: A project costing ₹ 20 lakhs yields annually a profit of ₹ 3 lakhs after depreciation @12.5% (straight line method) but before tax 50%. In this case cash inflow = Profit after tax + Depreciation = ₹ 3,00,000 – Tax ₹ 1,50,000 + Depre. ₹ 2,00,000 = ₹ 4,00,000 p.a.

$$\text{Payback period} = \frac{1,60,000 - 10,000}{15} \frac{\text{Cost of the project}}{\text{Annual cash inflow}} = \frac{20,00,000}{4,00,000} = 5 \text{ years.}$$

The Payback Reciprocal

A simple method of calculating the internal rate of return is the payback reciprocal which is 1 divided by the payback period.

Example: A project has an initial cash outlay of ₹ 2,00,000 followed by 10 years of annual cash savings of ₹ 50,000. The payback period is ₹ 2,00,000/₹ 50,000 = 4 years and the payback reciprocal is

$$\frac{1}{\text{Playback period}} = \frac{1}{4} = 25\%$$

A major drawback of the payback reciprocal that it does not indicate any cutoff period for the purpose of investment decision. It is, however, argued that the reciprocal of the payback would be a close approximation of the internal rate of return if the life of the project is at least twice the payback period and the project generates equal amount of the annual, cash inflows.

Accounting Rate of Return (ARR)

The accounting rate of return (ARR) method of evaluating capital budgeting projects is so named because it parallels traditional accounting concepts of income and investment. A project is evaluated by computing a rate of return on the investment, using accounting measures of net income. The formula for the accounting rate of return is:

$$\text{ARR} = \frac{\text{Annual revenue from project} - \text{Annual exp. of project}}{\text{Project investment}} \times 100$$

This rate is compared with the rate expected on other projects, had the same funds been invested alternatively in those projects. Sometimes, the management compares this rate with the minimum rate (called cut of rate) they may have in mind.

Merits: This method is quite simple and popular because it is easy to understand and includes income from the project throughout its life.

Net Present Value

- Net Present Value (NPV) refers to the difference between the present value of cash inflow minus cash outflows.
- NPV = Present value of cash inflow – Cash outflow

Decision Rules:

- Accept the project when NPV is positive, NPV > 0
- Reject the project when NPV is negative, NPV < 0
- May accept the project when NPV is zero, NPV = 0

Calculation and Decision Making

		Cash Flows	Present Value factor @ 10%	Present Value of cash inflow	Total Cash flows
A	Cash outflows	(100,000)	1.0		
B	Cash inflow				
	1st Year	45,000	0.909	40,905	
	2nd Year	40,000	0.826	33,040	
	3rd Year	35,000	0.751	26,285	
	4th Year	50,000	0.683	34,150	134,380

Since NPV = Present Value of cash inflow – Cash outflow

So, NPV = 134,380 – 100,000 = 343,80

The above calculation reveals that, the calculated NPV is positive. So, the project can be accepted

Pros and Cons of NPV

Pros

- NPV is used in capital budgeting to analyze the profitability of an investment or project
- Takes account of the time value of money
- Considers all the cash flows
- Provides better forecast

Cons

- It may not give reliable answers when desling with alternative projects under the conditions of unequal lives of project.
- It is very difficult to find and understand the concept of cost of capital.

Internal Rate of Return (IRR)

- IRR is that rate at which the sum of discounted cash inflows equals the sum of discounted cash outflows. It is the rate at which the net present value of the investment is zero.
- It is called intermal rate because it depends mainly on the outlay and proceeds associated with the project and not on any rate determined outside the investment.
- IRR decision rule: A project is acceptable if IRR>r

Calculation of IRR

		Cash flows	Present value factor @10%	Present value in cash inflow	Present value factor @30%	Present value in cash inflow
	Cash out follow = 100,000					
1	1st Year	45,000	0.909	40,905	0.7692	34,614
2	2nd Year	40,000	0.826	33,040	0.5917	23,668
3	3rd Year	35,000	0.751	26,285	0.4551	15,928
4	4th Year	50,000	0.683	34,150	0.3501	17,505

Calculation of IRR

Cash in follow - Cash out follow @10% = 134,380.00 – 100,000.00 = 34,380

Cash in follow – Cash out follow @30% = 91,715.00 – 100,000.00 = (8,285)

IRR = 10% + [34,380/{34/380 – (-8285)}] × (30% - 10%)

IRR = 10% + (34,380/42,665) × 20%

IRR = 10% + 0.8058 × 20%

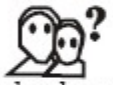
IRR = 26.11%

Internal Rate of Return (IRR)

Internal rate of return is the interest rate that discounts an investment's future cash flows to the present so that the present value of cash inflows exactly equals the present value of the cash outflows i.e., at that interest rate the net present value equals zero.

The discount rate i.e., cost of capital is considered in determination of the net present value while in the internal rate of return calculation, the net present value is set equal to zero and the discount rate which satisfies this condition is determined and is called Internal Rate of Return.

Any investment that yields a rate of return greater than the cost of capital should be accepted because the project will increase the value of the firm.



Did u know? Unlike, the NPV method, calculating the value of IRR is more difficult. The procedure depends on whether the cash flows are annuity (equal year wise) or non-uniform.

The following steps are taken in determining IRR for an annuity (equal cash flows):

1. Determine the payback period of the proposed investment.
2. From the table of Present value of Annuity, look for year that is equal to or closer to the life of the project.
3. From the year column, find two Present Value or discount factors closest to payback period, one larger and other smaller than it.
4. From the top row of the table note, the two interest rates corresponding to these Present values as in (3) above.
5. Determine IRR by interpolation

When cash flows are not uniform, an interest rate cannot be found using annuity tables.

Instead trial and error methods or a computer can be used to find the IRR. If the IRR is computed manually, the first step is to select an interest rate that seems reasonable (this can be done by calculating average annual cash flows by the annuity method as mentioned earlier) and then compute the present value of the individual cash flows using that rate.

If the net present value is positive, then the interest rate used is low, i.e., IRR is higher than the interest rate selected. A higher interest rate is then chosen and the present value of the cash flows is computed again. If the new interest rates yield a negative net present value, then a lower interest rate is to be selected. The process is repeated until the present value of cash inflow is equal to the present value of the cash outflows. Finding the rate of return using trial and error methods can be tedious, but a computer can accomplish the task quite easily.

Illustraton 1: A project costing ₹ 5,60,000 is expected to produce annual net cash benefits of ₹ 80,000 over a period of 15 years. Estimate the IRR. Also, find the payback period and obtain the IRR from it. How do you compare this IRR with the one directly estimated?

Hence from the present value of annuity 1—15 years closest factors to 7 are 7.191, (at 11% rate of discount) and 6.811 (at 12% rate of discount). Hence IRR would be somewhere between 11% and 12%.

Using interpolation IRR would be:

$$11\% + \frac{7.191 - 7}{7.191 - 6.811} = 11\% + \frac{0.191}{0.380} = 11.5\%$$

We know that reciprocal of payback period is a good approximation of the IRR provided the life of the project is large or at least twice the payback period and the project generates equal annual cash inflows. Since both the conditions are satisfied. IRR would be reciprocal of the payback period i.e., 1/7 = 14.28%.

The two IRR's are different. The second method is an approximation present value whereas the first gives the correct IRR, since at that discount rate cash inflows equals the cost of the project or the net present value is zero.

Pros and Cons of IRR

Pros

- Takes account of the time value of money
- Easy to be understood by managers
- Takes into account total cash inflows and total outflows

Cons

- Involves tedious calculations
- Difficult to use in choosing projects of varying sizes
- Difficult to choose when projects have the sam IRR
- Not dependent on the discount rate

Valuable Products are considering purchase of a machine for its production line. Two types of options are available deluxe model with ₹ 30,000 initial cost and economy model with ₹ 20,000 initial cost. Each model has 5 years life and no salvage value. The net cash flows after taxes associated with each investment proposal are:

	Deluxe Model	Economy Model
Net cash flows after taxes 1—5 years	₹ 9,000	₹ 6,000

Solution:

1. ***Net Present Value Method:***

		Cash inflow after taxes		Total Present Value	
Year's	Deluxe Model (₹)	Economy Model (₹)	PV Factor 10%	Deluxe Model (₹)	Economic Model (₹)
1 – 5	9,000	6,000	3.7907	34,116	22,744
		Deduct initial cost		30,000	20,000
		Net Present Value		4,116	2,744

Hence, the model that gives higher NPV should be chosen i.e. Deluxe Model.

Remark: Since capital outlay was higher for Deluxe Model it has given higher NPV.

2. ***Present Value Index:***

	Deluxe Model		Economy Model
=	$\frac{34,116}{30,000}$	=	$\frac{22,744}{20,000}$
=	1.1372	=	1.1372

Since both give same PI Index, we are indifferent as to both the models.

3. ***IRR:***

	Deluxe Model		Economy Model
=	$\frac{30,000}{9,000}$	=	$\frac{20,000}{6,000}$
=	3.33 years	=	3.33 years

Decision: If the capital is adequate there are no constraints, the proposal that gives higher NPV should be selected. In this case, the Deluxe Model.

9.4 Comparison – NPV and IRR Methods

Similarities

In respect of conventional and independent projects, the two methods give a concurrent acceptance-reject decision. In case of conventional investment cash outflows are confined to the initial

period (i.e. in the beginning) followed by cash inflows. The independent proposals refer to investment, the acceptance of which does not preclude the acceptance of others, so that all profitable proposals can be accepted and there are no constraints. The decision rule is that a proposal will be accepted if

1. NPV exceeds zero,
2. IRR exceeds the required rate of return.

Similarly, when NPV = Zero or the IRR = required rate of return, the project may be accepted or rejected.

Differences

In case of mutually exclusive capital project i.e., the acceptance of an investment precludes the acceptance of others (i.e. if there are alternative courses of action, only one can be accepted). Mutual exclusiveness of the investment projects may be technical or financial. Technical means projects with different profitabilities and selection of the more profitable. Financial means resource constraints, which is also called capital rationing.

The different ranking by NPV and IRR methods can be due to the following:

1. Size disparity
2. Time disparity
3. Unequal expected lives.

As stated earlier, the IRR criterion implicitly assumes that the cash flow generated by the project will be reinvested at the internal rate of return, as opposed to company's cost of capital in case of NPV. The assumption of the NPV method is considered to be superior since the rate can be consistently applied to all investment proposals.

Discounted Payback Period

- The discounted payback period is similar to the traditional payback period except that it uses discounted free cash flows rather then actual undiscounted cash flows
- The discounted Payback period is defined as the number of years needed to recover the initial cash outlay from the discounted free cash flows.

Profitability Index

- The profitability Index is the present value of an anticipated cash in flows divided by the initial investment. It is a method of assessing capital expenditure opportunities in the profitability index.

- Profitability Index (PI) = $\dfrac{\text{Present Value of cash inflows}}{\text{Present value of cash outflows}}$

This method is also called cost benefit ratio or desirability ratio

9.4.1 Net Present Value vs. Profitability Index

The investment proposal will be acceptable if:

1. PI is greater than one.

2. Positive Net Present Value.

Likewise, PI will be less than I when the investment proposal has a negative net present value under the NPV method.

Caution While evaluating mutually exclusive investment proposals, these methods will give different rankings. The best project is the one, which adds the most, among available alternatives, to the shareholders wealth. The NPV method by its very definition, will always select such projects.

9.4.2 Interrelation between Payback, Net Present Value, IRR and Profitability Index

We have seen $$\text{Payback period} = \frac{\text{Cost of the Project}}{\text{Annual cost saving/Inflows}}$$

We have also seen that investment proposal will be acceptable if PI is greater than one and Net Present Value is positive.

Again, NPV = 0, when the discount rate is equal to cost of capital and PI = I and by definition IRR is the interest rate that discounts an investments', future cash flows to the present so that present value of inflows equals to the present value of cash outflows i.e., NPV is equal to zero. Hence, under IRR, if interest rate is equal to cost of capital, NPV is zero and also PI = I.

The steps for determining IRR for an annuity (equal annual cash flows).

1. Payback period, which will give the cumulative present value factor.
2. From cumulative PV (discount) factor tables see the corresponding interest rate nearest to that figure corresponding to the life of the project (No. of years).

From the above discussion we can conclude the following:

1. Payback period = Cumulative Present value of Discount, based.
   (equal annual cash flows) on cost of capital.
2. At IRR rate of interest NPV = 0 i.e., Cost of the project = Cash outflows
   = Present value of cash inflows
   = Annual cash inflows × Cumulative discount factor
   (In case of equal annual cash inflows)

Example: Following are the data on a capital project being evaluated by the management of X Ltd.

	Project M
Annual cost saving	₹ 40,000
Useful life	4 years
IRR	15%
Profitability Index (PI)	1.064
NPV	?
Cost of capital	?
Cost of project	?
Payback	?
Salvage value	0
Find the missing values.	(Given cumulative PV 1-4 years @ 15% = 2.855)

Solution:

At 15% IRR, the sum total of cash inflows = Initial cash outlay i.e. cost of the project

Cumulative present value @ 15% for 4 years = 2.855 and Annual Cost saving ₹ 40,000

Hence, Total of Cash inflows = 40,000 × 2.855 = ₹ 1,14,200

Therefore, Initial Cost Outlay

i.e., Cost of the project = ₹ 1,14,200

$$\text{Payback period} = \frac{\text{Cost of the project}}{\text{Annual cost saving}} = \frac{1,14,200}{40,000} = 2.855 \text{ years}$$

$$= 2 \text{ years } 11 \text{ months}$$

$$\text{P I.} = 1.064$$

$$= \frac{\text{P.V. of cash inflow}}{\text{P.V. of cash outflow i.e. Cost of the project}}$$

$$\text{Hence, PV of Cash inflows} = 1.064 \times 1{,}14{,}200 = ₹\, 1{,}21{,}508.8$$

$$\text{NPV} = 1{,}21{,}508.8 - 1{,}14{,}200 = ₹\, 7{,}309$$

Current Present Value factor at Company's Cost of Capital

$$= \frac{\text{PV of Cash inflows}}{\text{Annual Cost Saving}}$$

$$= \frac{1{,}21{,}509}{40{,}000} = 3.0377$$

From the present value table corresponding to 4 years the discount/interest is 12%

$$\text{i.e., Cost of capital} = 12\%.$$

Example: Following are the data on a capital project with equal annual cash savings evaluated by the management of S company –

	Project
Cost	₹ 1,01,400
Payback	5.07 years
Annual Cost Saving	?
Useful life	?
Cost of Capital	?
NPV -	?
IRR	19 per cent
PI	1.14
Salvage Value	0

Solution:

1. Annual Cost Saving = Cost of Project/Payback period

= ₹ 1,01,400/5.07 = ₹ 20,000

2. At IRR rate of discount (i.e. 19%)

Cost of the project = PV of cash inflows

= ₹ 1,01,400

Hence, Cumulative Present Value at 19%

$$= \frac{1,01,400}{20,000} = 5.07$$

If we refer to Cumulative Present Value @19% Cumulative Present Value of 5.07 is computed at 19 years. Therefore, useful life =19 years.

3. PI = 1.14 at cost of capital rate of interest; at IRR rate of discount

PI index = 1. Hence Cumulative Present Value at cost of capital rate of interest = 5.07 × 1.14 = 5.778. By referring to Cumulative Present Value table up to 19 years. We find at 17% Cumulative Present Value 5,585 and at 16% = 5.877. Since 5,778 Cumulative Present value is lying between 5.877 and 5,585 by interpolation we get,

$$16 + \quad = \quad 16 + = 34 + 16 = 16.34\%$$

4. NPV at IRR rate of discount = 0 when PI = I

$$\text{Since PI} \quad = \quad 1.14$$

$$\text{Therefore, NPV} \quad = \quad 0.14 \times \text{Cost of the project} = 0.14 \times 1{,}01{,}400 = ₹1{,}41{,}196$$

9.4.4 Capital Rationing

The process of selecting the more desirable projects among many profitable investments is called capital rationing. Like any rationing it is designed to maximize the benefit available from using scarce resources. In this case the scarce resources are funds available for capital investments and the benefits are returns on the investments. The objective is to select the combination of projects, which would give maximization of the total NPV. The project selection under capital rationing involves two stages:

1. The identification of the acceptable projects,
2. To select the combination, of projects. The acceptability of projects can be based either on profitability/present value index or IRR.

There are various ways of resorting to capital rationing. For instance, a firm may effect capital rationing through budgets. Capital rationing may also be exercised by following the concept of "responsibility accounting", whereby management may introduce capital rationing by authorizing a particular department to make investment only up to a specified limit, beyond which the investment decisions are to be taken up by higher-ups.

In capital rationing, it may also be more desirable to accept small investment proposals than a few large investment proposals so that there may be full utilization of budgeted amount. This may result in accepting relatively less profitable investment proposals if full utilization of budget is a primary consideration.

Similarly, capital rationing may also mean that the firm foregoes the next most profitable investment following after the budget ceiling, even though it is estimated to yield a rate of return much higher than the required rate of return. Thus, capital rationing does not always lead to optimum results.

Example: S. Ltd. has ₹ 10,00,000 allocated for capital budgeting purposes. The following proposals and associated profitability indexes have been determined:

Project	Amount (₹)	Profitability Index
1	3,00,0.00	1.22
2	1,50,000	0.95
3	3,50,000	1.20
4	4,50,000	1.18
5	2,00,000	1.20
6	4,00,000	1.05

Which of the above investments should be undertaken? Assume that projects are indivisible and there is no alternative use of the money allocated for capital budgeting:

Solution:

We should go in for projects priority-wise based on PI Index:

Project	PI	Investment (₹)	Priority	Sum Total of Cash Inflows (₹)	NPV (₹)
1	1.22	3,00,000	I	3,66,000	66,000
3	1.20	3,50,000	2	4,20,000	70,000
5	1.20	2,00,000	2	2,40,000	40,000
4	1.18	4,50,000	3	5,31,000	81,000
6	1.05	4,00,000	4	4,20,000	20,000

Since there is no alternative use of money we should maximize capital spending provided project is profitable, that is, PI is more than I and NPV is positive. Combination of project 1, 3, 5 gives NPV of ₹ 1,76,000 but ₹ 1,50,000 remains unspent, whereas combination of 3, 4, 5 gives NPV of ₹ 1,91,000 and every amount is spent. Hence project 3, 4, 5 are to be taken.

9.4.5 Break-Even Time and Capital Budgeting for New Products

Time is being considered as a competitive weapon specifically for firms that bring a new product to market at a very fast rate in order to gain sizeable shares of total market sales for that product. Hence, the increased emphasis to the use of break-even time as a capital budgeting method and as a performance measures.

Break-Even Time (BET) is the time taken from the start of the project (the initial idea date) till the period the Cumulative Present Value of cash inflows of a project equal to present values of the total cash outflows. Hence, product proposals with shorter BET's are preferred to longer BETs if all other things are equal.

BET also promotes aggressive efforts by personnel on different functional areas such as product design and manufacturing to speed up the time taken to bring the product to the market and reap the benefits fast.

Example: Two products A and B are being evaluated by a Computer Manufacturing Company. The estimated cash outflows covering research and development, product design, manufacturing, marketing, distribution and customer services and cash inflows from sales are given below:

(₹ lakhs)

	Product A		Product B	
Year	Cash Outflows	Cash Inflows	Cash Outflows	Cash Inflows
1	8	0	10	4
2	6	14	7	32
3	22	34	17	26
4	13	37	6	8
5	10	22	0	2

The company uses a 14% required rate of return for discounting cash flows on a before tax basis.

Solution:

For Product A

(₹ lakhs)

Year	PV Factor	Cash Outflows	PV of cash Outflows	Cum PV of Cash Outflows	Cash Inflows	PV of cash Inflows	Cum. PV of cash Inflows.
1	0.877	8	7.016	7.016	0	-	0
2	0.769	6	4.614	11.63	14	10.766	10.766
3	0.675	22	14.85	26.48	34	22.95	33.716
4	0.592	13	7.696	34.176	37	21.904	55.62
5	0.519	10	5.19	39.366	22	11.418	67.038

For Product A, the present value of total cash outflows is ₹ 39.366 lakhs. At the end of 3 year, the cumulative present value of cash inflows is ₹ 33.716 lakhs and for 4th year the present value of cash inflows is ₹ 21.904 lakhs.

$$\text{BET of Product A} = 3 + \frac{39.366 - 33.716}{21.904} = 3.26 \text{ years}$$

For Product B

(₹ Lakh)

Year	PV Factor	Cash Outflows	PV of cash outflows	Cum PV of cash outflows	Cash inflows	PV of cash inflows	Cum. PV of cash inflows.
1	0.877	10	8.77	8.77	4	3.508	3.508
2	0.769	7	5.383	14.153	32	24.608	28.116
3	0.675	17	11.475	25.628	26	17.55	45.666
4	0.592	6	3.552	29.18	8	4.736	50.402
5	0.519	0	0	29.18	2	1.038	51.44

For Product B, the present values of the total cash outflows are ₹ 29.18 1akh. At the end of 2 year, the cumulative present value of cash inflows is ₹ 28,116 lakhs and for 3rd year the present value of cash inflows is 17.550.

$$\text{BET for Product B} = \frac{29.18 - 28.116}{17.550} = 2.06 \text{ years}$$

The High Peaks Sporting Goods Stores have been plagued by numerous burglaries over the last 3 years. To keep insurance premiums at reasonable level and protect ₹10,00,000 inventory, the store fixed a night watchman. The watchman has solved the burglary problem, but he costs the firm ₹12,000 a year.

He is occasionally absent from work due to sickness or bad weather. A security system company has offered to sell the store system that would eliminate the need for the night watchman. The system has an expected useful life of 15 years. The security system's salesperson is computing the cost of the system and will present a bid this week. The management estimates cost of capital at 16%.

Required:

1. What is the maximum bid the store should accept?
2. If the bid is ₹ 64,000 should the store accept?
3. If the actual life of the security system is 12 years instead of 15, does it have any effect on your answer in part (b)?

Solution:

In this case there are two alternatives:

1. To employ watchman at a salary of ₹ 12,000 a year.
2. To buy the system that has an expected life of 15 years.

If one buys the system it will save ₹ 12,000 per year for 15 years i.e., at zero date it is equivalent to 12,000 × Cum Discount factor at 16% for 1 – 15 years = 12,000 × 5.575 = ₹ 66,900. Hence the maximum bid the store should accept is ₹ 66,900. Any offer less than 66,900 is acceptable hence if the bid is ₹ 64,000 the store should accept.

If the actual life of the security system is 12 years, the saving is equivalent to 12,000 × Discount factors at 16% 1 – 12 years = 12,000 × 5,197 = ₹ 62,364. Hence the maximum should be restricted to ₹ 62.364 in this case. Therefore, the offer of ₹ 64,000 cannot be accepted in a situation where the life of security is 12 years.

Limitations

- There are certain factors like morale of the employees, good-will of the firm etc.' which cannot be correctly quantified but which otherwise substantially influence the capital decision.
- Urgency is another limitation in the evaluation of capital investment decisions.
- Uncertainty and risk pose the biggest limitations to the techniques of capital budgeting.

CAPITAL BUDGETING UNDER RISK & UNCERTAINITY

Risk and Uncertainty

- RISK AND UNCERTAINTY BOTH REFER TO SITUATIONS WITH MORE THAN ONE OUTCOME.
- HOWEVER, RISK DEFINES FUTURE EVENTS THAT CAN BE OBJECTIVELY SPECIFIED IN ADVANCE BASED ON PRIOR KNOWLEDGE;
- AN OBVIOUS EXAMPLE BEING THE THROW OF A DICE.
- *Uncertainty*, WHICH CHARACTERIZES MOST BUSINESS DECISIONS, RELATES TO EVENTS WHOSE PROBABILITIES CANNOT BE PREDICTED WITH ACCURACY.

Quantifying risk

- QUANTITATIVE TECHNIQUES TRANSFORM UNCERTAINTY TO *quasi-risk*, WHICH *assumes* A RANGE OF POSSIBLE OUTCOMES AND ASSIGNS SUBJECTIVE PROBABILITIES TO THE LIKELIHOOD OF EACH OCCURRING.

Types of Risks Facing Investments

- WE SHOULD ALSO NOTE THAT A PROJECT'S OVERALL UNCERTAINTY OR *total* RISK EMBRACES:
  - *Business* RISK THAT RELATES TO THE VARIABILITY OF FUTURE CASH FLOWS ARISING FROM AN INVESTMENT'S FUNDAMENTAL CHARACTERISTICS, AS WELL AS CHANGING ECONOMIC CONDITIONS.
  - *Financial* RISK ASSOCIATED WITH A PROJECT'S FUNDING AND HOW THE EARNINGS DISTRIBUTED TO INVESTORS DETERMINE THE COMPANY'S COST OF CAPITAL (DISCOUNT RATE).

Dysfunctional Risk Methodologies

- THE FOLLOWING RISK TECHNIQUES ARE POPULAR WITH MANAGEMENT. BUT UNFORTUNATELY THEY FAIL TO MAXIMISE SHAREHOLDER WEALTH.
  - *Modifications* TO THE CUT-OFF RATE FOR INVESTMENT THAT ADDS A *risk premium* TO THE DISCOUNT RATE.
  - *Point estimates* SUCH AS BEST, WORST OR MOST LIKELY NET CASH INFLOW;
    - *Minimax*, WHICH FOCUSES UPON THE *best* OUTCOME UNDER THE MOST *adverse* CONDITIONS;
    - *Laplace* CRITERIA, WHICH SELECT THE MOST FAVOURABLE *simple* AVERAGE OF A THREE POINT ESTIMATE;
    - *Probability* ESTIMATION WHICH APPLIES PROBABILITIES TO THREE POINT FORECASTS TO PRODUCE THE BEST *weighted* AVERAGE (SUBJECT TO THE PROVISO THAT THE SUM OF THE PROBABILITIES EQUALS ONE).

Decision Trees, Sensitivity and Computers

- *Decision trees* PROVIDE A *mind map* OF UNCERTAIN PROJECT CASH FLOWS BRANCH OUT FROM AN INVESTMENT (HENCE ITS NAME) AND MAY PROLIFERATE BEYOND A THREE-POINT ANALYSIS. *Conditional* PROBABILITIES ARE ATTACHED TO A SEQUENCE OF LIKELY FUTURE EVENTS. THE BRANCHES OF THE TRUNK ARISE FROM PREVIOUS MANAGERIAL DECISIONS (CONTROL FACTORS) AND CHANCE (UNCONTROLLABLE FACTORS).
-
- *Sensitivity analysis* DECONSTRUCTS CASH DATA THAT COMPRISE AN INITIAL NPV COMPUTATION INTO ESTIMATES OF ITS COMPONENT PARTS. EACH VARIABLE IS THEN ANALYSED SEQUENTIALLY, USING *partial equilibrium analysis*. BY HOLDING ALL OTHER VARIABLES CONSTANT AND GAUGING THE IMPACT ON THE APPROPRIATE INVESTMENT CRITERIA OF PERCENTAGE CHANGES TO THE VARIABLE UNDER OBSERVATION, ITS CRITICAL VALUE IS ESTABLISHED.

Mean-Variance Methodology

- FORECAST DATA THAT EXTENDS BEYOND POINT ESTIMATIONS TO MULTI-VALUED OUTCOMES MAY BE CONVERTED TO QUASI-RISK USING THE MORE SOPHISTICATED TECHNIQUE OF *mean-variance analysis*.
- BASED ON CLASSICAL PROBABILITY THEORY, MANAGEMENT ASSUME CASH FLOWS ARE *random* VARIABLES, WHICH CONFORM TO A *normal* DISTRIBUTION WITH A *symmetrical* BELL-SHAPED CURVE AS FOLLOWS:

Calculating Mean- EMV

- THE *mean* IS DERIVED BY FIRST MULTIPLYING A SPECTRUM OF ANNUAL CASH FLOWS C_I BY RESPECTIVE PROBABILITIES P_I (SUBJECT TO THE PROVISO THAT $\sum P_I = 1.0$). THEN THE PRODUCTS $C_I P_I$ FOR ANY NUMBER OF CASH FLOWS (N) ARE SUMMATED TO DERIVE AN *expected monetary value* (EMV) AT TIME PERIOD T:

- (1) $EMV_T = \Sigma C_I P_I$

  - N

  - T=I

Calculating ENPV

- NEXT, THE ANNUAL EMV SERIES IS DISCOUNTED OVER THE APPROPRIATE PERIODS AT A *risk-free* RATE (AVOIDING DOUBLE-COUNTING) TO DETERMINE ITS EXPECTED PV, (EPV).
- FROM THIS WE SUBTRACT THE INVESTMENT COST, $I_{O,}$ TO OBTAIN A PROJECT'S *expected* NPV, (ENPV) IN THE USUAL MANNER:

- $ENPV = [\ \Sigma\ EMV_T / (1+R)^T\ ] - I_0 = EPV_N - I_0$

Use of Standard Deviation or Variance

- OBVIOUSLY, EMV TIME-SERIES AND ENPV ANALYSES IMPROVE UPON POINT ESTIMATES.
- BUT PROJECT SELECTION USING ENPV MAXIMISATION ALONE CANNOT MINIMISE RISK BECAUSE IT DOESN'T CALIBRATE THE DEGREE TO WHICH CASH FLOWS VARY AROUND THEIR MEAN (BUSINESS RISK) OR MANAGERIAL REACTION TO THIS VARIABILITY.
- TO RESOLVE THE DILEMMA, THE *standard deviation* IS USED TO MEASURE THE *average* DISPERSION OF CASH FLOWS FROM THEIR EMV.
- MANAGEMENT THEN COMPARE THE STANDARD DEVIATION WITH THE EXPECTED RETURN TO ASSESS A PROJECT'S RISK-RETURN PROFILE; THE INTERPRETATION BEING THAT FOR A GIVEN RETURN, THE LOWER THE STANDARD DEVIATION THE LOWER THE RISK AND *vice versa*.

Calculation of Mean-Variance Analysis

C_i	P_i	C_iP_i	$(C_i - EMV)^2$	P_i	$(C_i - EMV)^2 P_i$
£		£			
8	0.1	0.80	3.61	0.1	0.361
7	0.2	1.40	0.81	0.2	0.162
6	0.4	2.40	0.01	0.4	0.004
5	0.3	1.50	1.21	0.3	0.363
				(1.0)	
EXPECTED MONETARY VALUE (EMV) £6.10			VARIANCE (VAR = σ^2) = 0.890		
			S.D. ($\sqrt{VAR} = \sigma$) = £0.943		

Z statistics

- REFER BACK TO THE FIGURE WHICH SKETCHED THE *area under the standard normal curve* AND THE PROBABILITY THAT A VARIABLE'S VALUE LIES WITHIN A NUMBER OF STANDARD DEVIATIONS AWAY FROM THE MEAN.
- BECAUSE THESE PROBABILITIES ARE *the same* FOR ANY NORMAL DISTRIBUTION THEY HAVE LONG BEEN QUANTIFIED IN TABLES BASED ON THE *z statistic,* WHICH STANDARDISES ANY VARIABLE'S ACTUAL DEVIATION FROM THE MEAN BY REFERENCE TO THE STANDARD DEVIATION.

Calculating the z score

- FOR A PARTICULAR CASH FLOW (C_l) DRAWN FROM A DISTRIBUTION WITH KNOWN MEAN AND VARIANCE:

- **$Z = C_l - EMV / \sigma (C_l)$**

Using the z-score table

- WE THEN CONSULT THE TABLE TO ESTABLISH THE AREA UNDER THE NORMAL CURVE BETWEEN THE RIGHT *or* LEFT OF Z (PLUS OR MINUS).
- THE VALUES SO YIELDED ESTIMATE THE PROBABILITY THAT THE EXPECTED CASH FLOW WILL BE A GIVEN NUMBER OF STANDARD DEVIATIONS AWAY FROM THE MEAN.
- SINCE A NORMAL DISTRIBUTION IS SYMMETRICAL, THE PROBABILITY OF A VARIABLE DEVIATING ABOVE *and* BELOW THE MEAN IS GIVEN BY 2Z (TWO-TAILED Z).

Finding z-scores

- TO DETERMINE THE PROBABILITY OF CONTRIBUTIONS DEVIATING ABOVE OR BELOW THE MEAN AS SPECIFIED, WE MUST FIRST CALCULATE THE FOLLOWING Z STATISTICS USING THE EQUATION ABOVE FOR CASHFLOWS RS 6.10, RS 6.50, RS 5.50 AS FOLLOWS:
-
- RS 6.10 IS (6.10 - 6.10) / 0.943 = ZERO σ FROM THE MEAN (OBVIOUSLY)
-
- RS 6.50 IS (6.50 - 6.10) / 0.943 = +0.42 σ FROM THE MEAN
-
- RS 5.50 IS (5.50 - 6.10 / 0.943 = -0.64 σ FROM THE MEAN

- NEXT WE CONSULT THE TABLE FOR THE AREA UNDER THE STANDARD NORMAL CURVE WHERE *z* EQUALS ZERO, 0.42 AND 0.64 (AT 5% SIGNIFICANCE).THE *mean-z* AREAS ARE 0.1628 AND 0.2389 RESPECTIVELY
- THUS, THE *total* AREA UNDER THE CURVE, BETWEEN +0.42 AND - 0.64, EQUALS 0.4017.
- SO, THERE IS A ROUGHLY A 40 PERCENT PROBABILITY OF THE CONTRIBUTION RANGING FROM RS 6.50 TO RS 5.50.

Nature of Risk

In the context of capital budgeting, the term, risk, refers to the chance that a project will prove unacceptable – that is NPV < ₹ 0 or IRR < cost of capital. More formally, risk in capital budgeting is the degree of variability of cash flows. Projects with a small chance of acceptability and a broad range of expected cash flows are more risky than projects that have a high chance of acceptability and a narrow range of expected cash flows.

In the capital budgeting projects, risk stems almost entirely from cash inflows, because the initial investment i.e., cash outflow is generally known with relative certainty. These inflows derive from a number of variables related to revenues expenditures and taxes.

COMMONLY USED TECHNIQUES

- CAPITAL BUDGETING TECHNIQUES UNDER RISK AND UNCERTAINITY

9.7 Conventional Techniques to Handle Risk

The following are conventional techniques to handle risk in capital budgeting:

- Payback
- Risk adjusted discount rate
- Certainty equivalent

9.7.1 Payback

Payback is one of the oldest and commonly used methods for explicitly recognizing risk associated with an investment project. Business firms using this method usually prefer short payback to longer one and often establish policies that a firm should accept guidelines with some maximum payback period say three to five years. Apart from simplicity, payback makes an allowance for risk by:

1. Focusing attention on the near term future and thereby emphasizing liquidity through early recovery of capital and
2. By favouring short-term projects over long-term riskier projects.

9.7.2 Risk Adjusted Discount Rate Approach (RAD)

Under this method, the amount of risk inherent in a project is incorporated in the discount rate employed in the present value calculations. The relatively risky projects (e.g. project involving introduction of new product into the untried market) would have relatively high discount rates and relatively safe projects would have relatively low discount rates. The rationale for using different risk adjusted rates for different projects is as follows. The rate of discount or the cost of capital is the minimum acceptable rate of return which the investors demand in providing capital to the firm for that type of investment since such rate is applicable elsewhere in the economy on assets of similar risk. If the project earns less than the rates earned in the economy for that risk, the shareholders will earn less and the value of the company's shares will fall. A well accepted economic premise is that the required rate of return should increase with increase in risks. Hence, the greater the risk, the greater should be the discount rate and vice versa.

The risk-adjusted rate can be used with both the NPV and IRR methods of evaluation of capital expenditure. If NPV were positive, the proposal would qualify for acceptance. In case of the IRR, as a decision criterion, the internal rate of return would be compared with the risk adjusted required rate of return and if the former exceeds the latter, the proposal would be accepted, otherwise not.

The risk in connection with future projections has two dimensions. First as already mentioned, riskiness of the projects at a particular point of time became of the nature of proposals, e.g., expansion of new products. Second, the risk may be different in the case of the same project over time e.g., risk at the end of Second year may be more than that at the end of first year.

Advantage

1. This method is simple to calculate and easy to understand, since companies in actual practice apply different standards of discount for different projects.

Disadvantages and Difficulties

1. Difficulty encountered is how to express a higher risk in terms of higher discount rates. It is doubtful if the exercise would give objective results.
2. It does not make direct use of the information available from the probability distribution of expected future cash. Conceptually, this approach adjusts the wrong element. It is the future cash flow of a project, which is subject to risk and hence should be adjusted and not the required rate of return.
3. The process of adding the risk premium to the discount rate leads to compounding of risk over time. In other words, this method implies increase of risk with time and therefore proposal in which risk does not necessarily increase with the time may not be properly evaluated by this method.

Example: Let us determine the risk adjusted net present value of the following:

	A	B	C
Net cash outlays (₹)	1,00,000	1,20,000	2,10,000
Project life	5 years	5 years	5 years
Annual cash inflow (₹)	30,000	42,000	70,000
Co-efficient of variation	0.4	0.8	1.2

The company selects the risk-adjusted rate of discount on the basis of coefficient of variation:

Coefficient of variation	Risk adjusted rate of discount	PV factor 1 to 5 years at risk adjusted rate of discount
0.00	10%	3.791
0.40	12%	3.605
0.8	14%	3.433
1.2	16%	3.274
1.6	18%	3.127
2.0	22%	2.864
More than 2.0	25%	2.689

Solution:

Project	Net cash outflow ₹	Coefficient of variation	Market discount rate	Annual cash inflow ₹	PV Factor (1-5 years) at market discount Rate	Discounted cash inflow ₹	NPV
A	1,00,000	0.4	12%	30,000	3,605	1,08,150	8,150
B	1,20,000	0.8	14%	42,000	3,433	1,44,186	24,186
C	2,10,000	1.2	16%	70,000	3,274	2,29,180	19,180

9.7.3 Certainty Equivalent Approach

Under this method, risk element is compensated by adjusting cash inflows rather than adjusting the discount rate. The risk adjustment factor is expressed in terms of certainty – equivalent coefficient i.e. the relationship between certain (riskless) cash flows and risky (uncertain) cash flows. The certainty equivalent coefficient can assume a value between 0 and 1 and is inversely related with risk. If risk is more, certainty is less and certainty coefficient small and vice-versa. The coefficients can be determined by subjective or objective assessments of cash flows that will rise certainly and cash flows that are likely to occur.

The second step under this approach after conversion of expected cash flows into certainty equivalents, is to calculate their present values based on the risk-free rate of discount (which appropriately reflects the time value of money). Finally, it has to be decided whether the project would be accepted or not, based on either NPV or the IRR method.

Advantages

1. It is simple to calculate.
2. It incorporates risk by modifying the cash flows, which are subject to risk.

Conceptually, it is superior to the time adjusted discount rate approach.

Weakness and Difficulties

1. Being a subjective estimate it cannot be objective, precise and consistent, hence conclusions based on such estimates are open to question.
2. It does not directly use the probability distribution of possible cash flows.
3. It cannot be consistently applied to various projects and over time.

Example:

		₹
Cash outflows		1,50,000
Cash inflows	Year 1	70,000
	Year 2	90,000
	Year 3	60,000

Riskless rate of return 9%

Risk adjusted rate of return for the current project 20%

Certainty equivalent coefficients for future cash inflows:

Year 1	0.90
Year 2	0.80
Year 3	0.65

Solution:

NPV based on risk-adjusted rate of discount

$$= -150{,}000 + \frac{70{,}000}{1.20} + \frac{90000}{(1.20)^2} + \frac{60000}{(1.20)^3}$$

$$= -150{,}000 + 58{,}333 + 62500 + 41667 = -150000 + 162500$$

$$= 12500, \text{ positive; hence project should be accepted}$$

NPV based on certainty equivalent coefficient:

$$= -150{,}000 + \frac{70{,}000 \times 0.90}{1.09} + \frac{90000 \times 0.80}{(1.09)^2} + \frac{60000 \times 0.65}{(1.09)^3}$$

$$= -150000 + 57798 + 60601 + 30115$$

$$= -150000 + 148514 = -1486$$

$$= \text{Negative; hence project should not be accepted.}$$

Hence from the above illustration, it is clear that both the above methods may not yield identical results.

9.7.4 Probability Distribution Approach

The probability distribution of cash flows over different periods provides valuable information about expected value of return and the dispersion of probability distribution of possible returns. On this basis, an accept-reject decision can be taken.

The application of probability distribution approach in analysing risk in capital budgeting depends upon the behaviour of the cash flows whether the cash flows are (a) independent or

Notes

(b) dependent. Independent means cash flows in future years are not affected by cash flows in the preceding or following years. On the other hand, when cash flows in one period are dependent on cash flows in previous year, they are referred to as dependent cash flows. Let us first discuss the application of probability theory to analyze risk in capital budgeting assumption of independent cash flows:

1. $$\text{NPV} = \sum_{t=1}^{n} \frac{\text{Expected cash inflow during each period}}{(1+\text{riskless rate of interest})} - \text{Cash outflow}$$

$$t = 1\ (1 + \text{riskless rate of interest})t$$

$$t = \text{years/period}$$

Now expected cash inflow in each period = Diff. levels of cash inflow × Probability at each level

2. $$\sum_{t=1}^{n} \frac{\text{Probability at each level} \times \left(\text{different levels cash inflow in each period} - \text{expected cash inflow each period}\right)^2}{(1+\text{riskless rate of interest})^{2t}}$$

3. By making use of normal probability distribution, one can analyze further. The element of risk in capital budgeting i.e. probability of different expected values of NPV i.e. the probability, of NPV having the value, zero or less, greater than within the range of two values and so on. Thus, the normal probability distribution an important statistical technique in the hands of decision makers for evaluating the extent of risk of the project.

Notes The normal probability has a number of useful properties as follows:

1. The area under normal curve, representing the normal probability distribution equal to 1 (0.5 on either side of the mean).
2. The curve has its maximum height at its expected value i.e., mean.
3. The probability of occurrence beyond 3 x Standard Deviation is very near zero (0.26%).
4. Probability of an outcome falling within plus or minus 1 × standard deviation from the mean is 0.6826 or 68.26%, Range $\pm$ 2 standard deviation 95.46%. Range $\pm$ 3 standard deviation 99.74%.

Example: A company is considering an investment in a project requiring initial outlay of ₹ 50,000 with expected cash inflow generated over 3 years as follows:

Year 1		Year 2		Year 3	
Cash flow (₹)	Probability	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
15000	0.2	20000	0.5	25000	0.1
20000	0.4	23000	0.1	30000	0.3
25000	0.3	25000	0.2	35000	0.3
30000	0.1	28000	0.2	50000	0.3

1. Assuming the probability distributions of cash outflows for future periods are independent, the firm's cost of capital is 10% and the firm can invest in 5% treasury bills, determine the expected NPV.

2. Determine the standard deviation about the expected value.

3. If the total distribution is approximately normal and assumed continuous.

   (a) What is the probability of the NPV being zero or less.

   (b) Greater than zero.

   (c) Profitability index being 1.00 or less.

   (d) At least equal to mean.

   (e) 10% below mean and 10% above mean.

   (f) The probability of NPV being (a) between the range of ₹15000 and ₹ 25,000 (b) between the range of ₹10000 and ₹20, 000 (c) at least ₹35, 000 (d) at least ₹ 7000.

Solution:

Period I			Period II			Period III		
Cash inflow (₹)	Probability	Cash inflow × probability (₹)	Cash inflow (₹)	Probab-ility	Cash inflow × probability (₹)	Cash inflow (₹)	Probab-Ility	Cash inflow × probability (₹)
15000	0.2	3000	20000	0.5	10000	25000	0.1	2500
20,000	0.4	8000	23000	0.1	2300	30,000	0.3	9,000
25,000	0.3	7500	25000	0..2	5000	35,000	0.3	10,500
30,000	0.1	3000	28000	0.2	5600	50,000	0.3	15,000
	Mean	21,500			22,900			37,000

Determination of NPV

Mean of Cash Inflow		PV factor @ 5% (riskless)	Total PV (₹)
Period 1	21500	0.952	20468
Period 2	22900	0.907	20770
Period 3	37000	0.864	31968
			73206
Less Cash Outflow			50000
NPV			23206

Determination of standard deviation of each period:

Period 1		Square of Deviation of Mean	Probability	Square of Deviation of Mean × Probability
Cash inflow (₹)	Deviation from Mean			
15000	-6500	42250000	0.2	8450000
20000	-1500	2250000	0.4	900000
25000	3500	12250000	0.3	3675,000
30,000	8500	72250,000	0.1	7225,000
				20250000

Notes

Standard Deviation = $\sqrt{10290000}$ = 4500

Period 2		Square of Deviation of Mean	Probability	Square of Deviation of Mean × Probability
Cash inflow ₹	Deviation from mean			
20000	–2900	8410000	0.5	4205000
23000	100	10000	0.1	1000
25000	2100	4410000	0.2	882000
28000	5100	26010000	0.2	5202000
				10290000

Standard Deviation = $\sqrt{20250000}$ = 3208

Period 3		Square of Deviation of Mean	Probability	Square of Deviation of Mean × Probability
Cash inflow ₹	Deviation from Mean			
25000	–12000	144000000	0.1	14400000
30000	–7000	49000000	0.3	14700000
35000	–2000	4000000	0.3	1200000
50000	+13000	169000000	0.3	50700000
				81000000

Standard Deviation = $\sqrt{810,00,000}$ = 9000

Standard Deviation about expected values:

$$\frac{189,850}{4.355} = ₹\ 43594\sqrt{\frac{(4500)^2}{(1.05)^2}+\frac{(3208)^2}{(1.05)^4}+\frac{(9000)^2}{(1.05)^6}}$$

Probability of NPV being zero or less: Calculate the difference between the specified point and NPV and then divide by standard deviation (NPV). This is referred to as Z. In this case $Z = \frac{0 - 23206}{9342} = -2484$. According to Table Z, the Probability of NPV being zero is 0.4934. but the area. For the area of the left hand side of the normal curve is equal to 0.5, the probability of he NPV being zero or less would be 0.5 – 0.4934 i.e. 0.0066. It means there is 0.66% probability that the NPV of the project will be zero or less.

Greater than zero = 100 – 0.66% (as per above) = 99.34%

Probability index being 1.00 or less: For PI Index to be 1.00 or less, the NPV would have to be zero or negative. Thus, the probability would be equal to 0.66% as calculated in the earlier part.

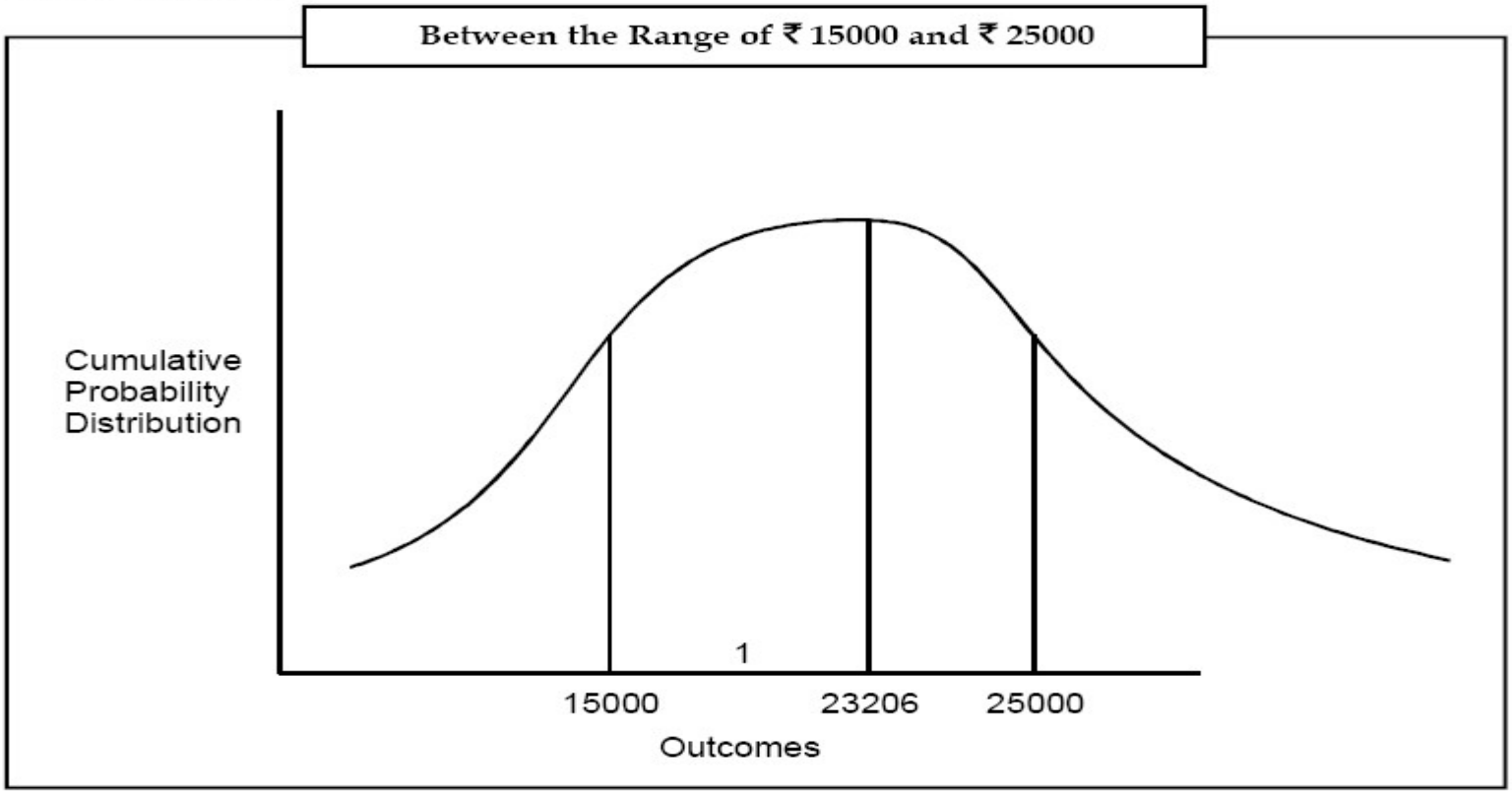
Atleast equal to mean: i.e. $Z = \frac{23206 - 23206}{9342} = 0$

Reading from the normal distribution table, we get the probability corresponding to 0 as 0. Therefore, the probability of having NPV at least equal to mean would be equivalent to the area to the right of the curve i.e., 0.5 = 50%.

10% below mean, and 10% above mean:

$$Z = \frac{20,885 - 23206}{9342} = -0.248$$

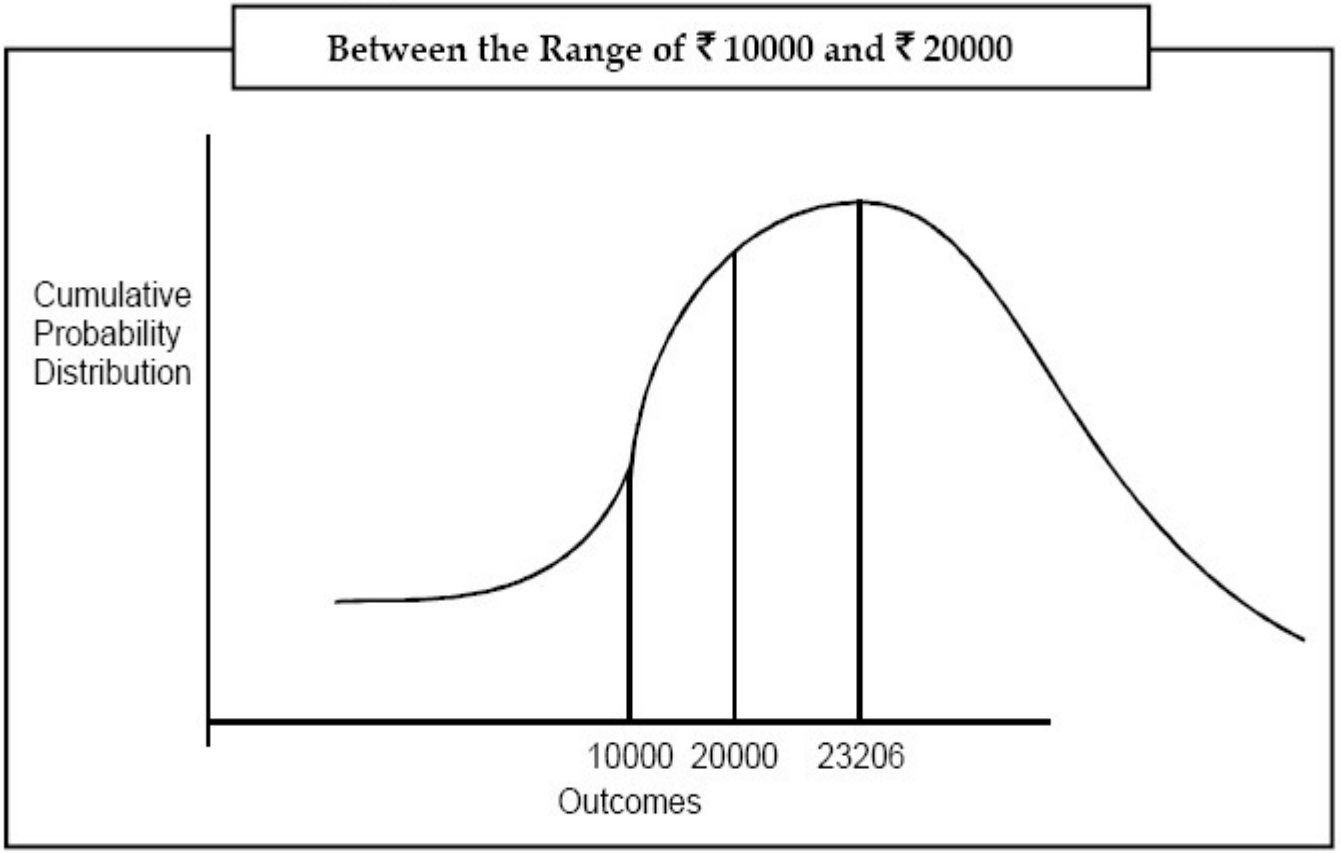
According to Table Z, the probability comes to 0.0979 i.e., 9.79% similarly for 10% above mean it comes to 9.79%.



$$Z_1 = \frac{15000 - 23206}{9342} = -0.878$$

$$Z_2 = \frac{25000 - 23206}{9342} = -0.192$$

According to Table Z, the probability corresponding to the respective values of Z1 and Z2 are 0.3100 and 0.0761. Summing up the values we have 0.3861 i.e., 38.61%.



$$Z_1 = \frac{10000 - 23206}{9342} = -1.414$$

$$Z_2 = \frac{20000 - 23206}{9342} = -0.343$$

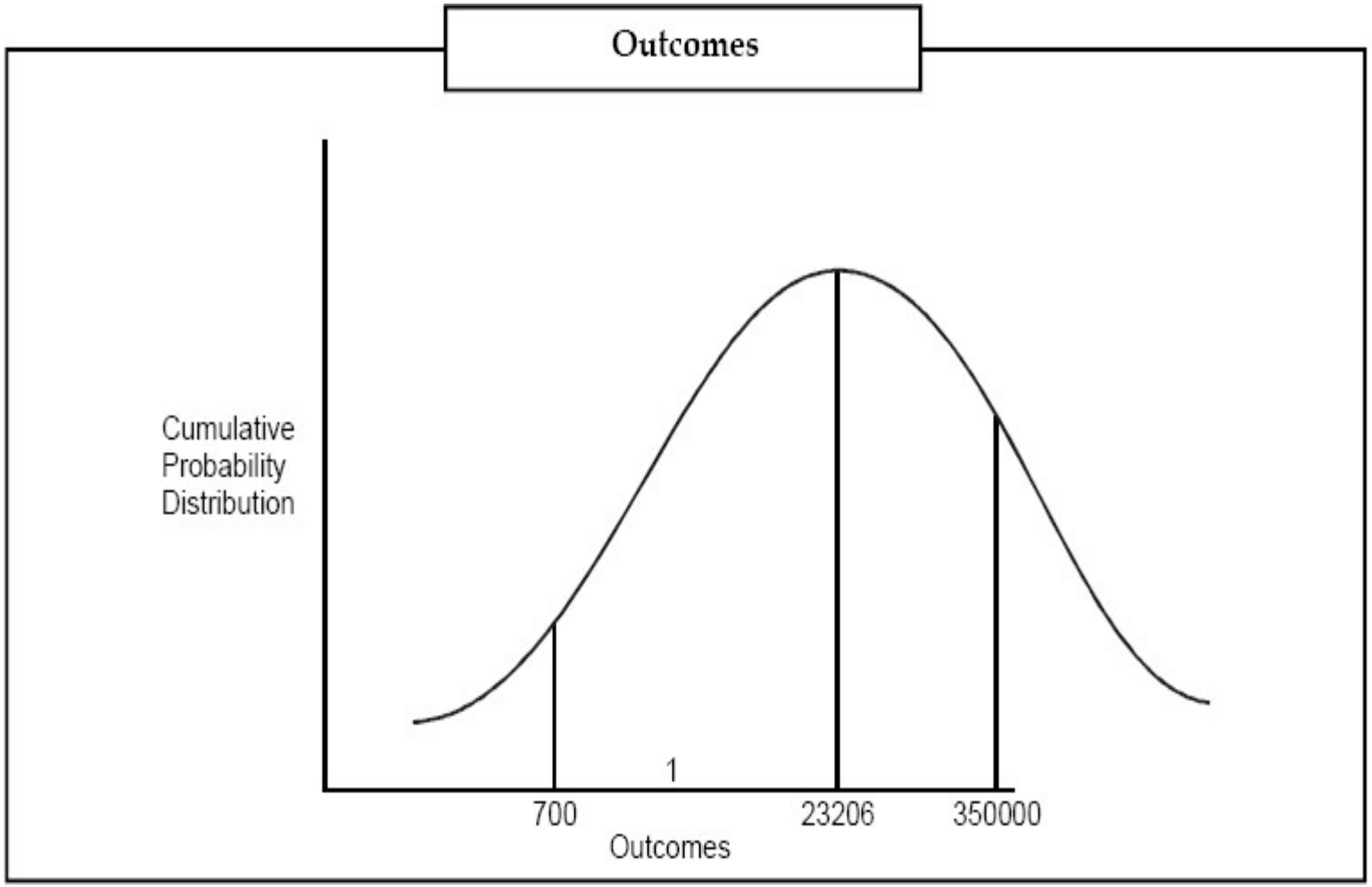
According to Table Z, the probability corresponding to these values of Z is 0.4213 and 0.1341. Since both are on the left side of the normal curve, the probability of having its value between ₹ 10,000 and ₹ 20,000 would be the difference i.e. 0.4213 – 0.1341 = 0.2872 i.e., 28.72%.

At least ₹ 35,000 $Z = \frac{35000 - 23206}{9342} = 1.262$

According to table Z, the probability of having the NPV values ₹ 35,000 is 0.3965. The probability of having NPV ₹ 35,000 or more would be 0.5 – 0.3965 = 0.1035 or 10.35%.

At least ₹ 7000 in this case $Z = \frac{7000 - 23206}{9342} = -1.735$

According to Table Z, the probability of having the NPV value of ₹ 7000 is 0.4586. The probability of NPV at least equal to ₹ 7000 would be more by 0.50 (area to the right side of the mean) i.e. 0.9586 or 95.86%.


Outcomes
Cumulative
Probability
Distribution
1
700
23206
350000
Outcomes

9.7.5 Decision-Trees Approach (DT)

DT is another useful alternatives for evaluating risky investment proposals. Under this method, every possible outcome is weighed in probabilistic terms and then evaluated. This approach is especially useful for situations in which decisions at one point in time also affect the decisions of the firm at some later date. Another useful application of this approach is for such projects, which require decisions to be made in sequential parts.

A decision tree approach as the name suggests, is a pictorial representation in tree form along with branches of the magnitude, probability and inter relationship of all possible outcomes. As a matter of convention the decision point is denoted by the symbol and the evens are denoted by 0. The key steps in decision tree analysis are:

1. Identification of the problem and alternatives
2. Delineation of the decision tree
3. Specification of probabilities and monetary outcome
4. Evaluation of various decision alternatives

Example: A firm has an investment proposal, requiring an outlay of ₹ 40,000. The investment proposed is expected to have 2 years' economic life with no salvage value. In year I, there is a 0.4 probability that cash inflow after tax will be ₹ 25000 and 0.6 probability that cash inflow after tax will be ₹ 30,000. The probabilities assigned to cash inflows after tax for the year II are as follows:

The cash inflow year I	₹ 25,000		₹ 30,000	
The cash inflow year II	₹ 12000	Probability 0.2	₹ 20,000	Probability 0.4
	₹ 16,000	Probability 0.3	₹ 25,000	Probability 0.5
	₹ 22,000	Probability 0.5	₹ 30,000	Probability 0.1

The firm uses a 10% discount rate for this type of investment.

Required:

1. Construct a decision-tree for the proposed investment project.
2. What net present value will the project yield if worst outcome is realized? What is the probability of occurrence of this NPV?
3. What will be the best and probability of that occurrence?
4. Will the project be accepted? 10%, Discount factor 1 year 0.909
   2 year 0.826

Solution:

Investment proposal, capital Outlay ₹ 40000

Year 1 prob	Cash (nflow (-₹)	Year 2 prob.	Cash Inflow (₹)	Path no	Expected NPV at 10% rate of discount	Joint prob (prob year1x prob. Year 2)	Expected NPV x Joint Prob.
0.4	25,000	0.2	12,000	1	-7363	0.08	-589
		0.3	16,000	2	-4059	0.12	-487
		0.5	22,000	3	897	0.20	179
0.6	30,000	0.4	20,000	4	3,790	0.24	910
		0.5	25,000	5	7,920	0.30	2,375
		0.1	30,000	6	12,050	0.06	723
							3112

Expected NPV:

	Cash Inflow Year 1	×	Discount Factor	+	Cash inflow Year 2	×	Discount Factor	–	Cash Outlay	
Path 1 =	25000	×	0.909	+	12000	×	0.826	–	₹ 40000	= 32637 – 40000 = – 7363
2 =	25000	×	0.909	+	16000	×	0.826	–	₹ 40000	= 35941 – 40000 = – 4059
3 =	25000	×	0.909	+	22000	×	0.826	–	₹ 40000	= 40897 – 40000 = 897

4 =	30000	×	0.909	+	20000	×	0.826	– ₹ 40000	= 43790 – 40000 = + 3790
5 =	30000	×	0.909	+	25000	×	0.826	– ₹ 40000	= 47920 – 40000 = 7920
6 =	30000	×	0.909	+	30000	×	0.826	– ₹ 40000	= 52050 – 40000 = 12050

The last column shows the expected NPV, which is obtained by summing up the product of NPV and corresponding joint probability. The term of these weighted NPV is positive and therefore, the project would be accepted.

This DT approach has the advantage of exhibiting in a bird's eye view of all the possibilities associated with the proposed project. Management also becomes aware well in advance of the adverse possibilities when NPV is negative. The conditional nature of cash inflow associated with the project is clearly shown. The main limitation of the method is that decision format may become itself complex and difficult to understand and construct if the number of years of the expected life of the project and the number of possible outcomes for each year are of large. For instance, if we have 3-year project, there will be 27 paths (i.e. 33) and 59,049 (31^{11}) paths if the project life is 10 years, assuming only three possible outcomes.

9.7.6 Applied Utility Theory

If the risk factor of the top management (i.e. the group that makes decisions regarding investment) can be translated into a utility curve, the risk factor can be incorporated in the final decision in a fairly simple and consistent manner.

Did u know? A utility function describes a person's risk attitude.

The utility function shows the utilities (an arbitrary measure of satisfaction) associated with different monetary outcomes. Normally, utilities are shown as the vertical axis and monetary returns are shown on the horizontal axis. A person's utility function can be obtained as follows:

The utility function shows the utilities (an arbitrary measure of satisfaction) associated with different monetary outcomes. Normally, utilities are shown as the vertical axis and monetary returns are shown on the horizontal axis. A person's utility function can be obtained as follows:

1. Find the monetary incomes that a person considers as relevant. The end may be designated as A and B.

2. Assign utility of 0 to A (the lowest outcome) and utility of 1 to B (the highest outcome).

3. Determine the utility associated with any intermediate value by the following process:

   (a) Present the individual with two alternatives. The first alternative of the intermediate value X, the utility of which is to be determined, second alternative consists of a lottery which has two possible outcome A and B with probabilitiesP1 and P2 attached to them (P1 + P2 = 1). Ask the individual to choose one of the alternatives.

(c) Continue the revision process laid down in (b), till the individual indifferent to both alternatives.

(d) When the individual becomes indifferent to both alternatives, the utility of A is simply the value of P2.

4. Obtain the utility values of several intermediate outcomes and plot the utility function.

Once the utility curve of the decision-making unit is obtained, the expected utility project is measured as follows:

1. Define the probability distribution of NPV.

2. Calculate the expected utility by using utility function.

$\Sigma u\ (NPVi)\ Pi$

where $u(NPV_i)$ = utility of the ith positive NPV

P_i = Probability of ith possible outcome

Let the utility for money for the decision maker be as follows:

Example: Let the probability distribution of NPV for a project be as follows:

NPV	-10,000	0	20,000	30,000
Probability	0.1	0.3	0.4	0.2

Money (₹)	Utility
– 20,000	0
– 10,000	0.10
0	0.35
+ 10,000	0.50
+ 20,000	0.68
+ 30,000	0.85
+ 40,000	0.90
+ 50,000	1.00

Hence the expected utility of the project is:

$$= 0.10 \times 0.1 + 0.35 \times 0.3 + 0.68 \times 0.4 + 0.85 \times 0.2$$

$$= .01 + 0.105 + 0.272 + 0.17 = 0.557$$

Evaluation: This method is superior to other methods of risk adjustments since it offers an approach for incorporating the risk factor consistently. However, the following are the problems:

1. It is difficult to obtain the utility functions of an individual i.e., translating the risk attitude of an executive requires patience and ingenuity on the part of the analyst.
2. Utility function of the decision-maker may not remain stable over time, since it is dependent on the organization financial position.
3. There is no acceptable way of determining the utility function of a group. (Since investment decisions are group decisions)

Hence, utility theory is not very useful for investment decisions where the cost and benefits are spread over long period. It is, however, a potential tool for short-term investment.