

(b) Find the singular solution of the differential equation

$$y = px + \sqrt{a^2 p^2 + b^2}, \quad p = \frac{dy}{dx}.$$

2017

MATHEMATICS

[Generic Elective]

(CBCS)

[First Semester]

PAPER – GE1T

Full Marks : 60

Time : 3 hours

The figures in the right-hand margin indicate marks

UNIT – I

(Calculus - I)

1. Answer any three questions : 2 x 3

(a) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$. 2

(b) Find the envelope of the straight line $y = mx + \frac{a}{m}$, m being the variable parameter ($m \neq 0$). 2

- (c) Find the asymptotes of the curve $y = xe^{\frac{1}{x}}$. 2
- (d) Find the point(s) of inflexion on the curve $x = (y-1)(y-2)(y-3)$. 2
- (e) State Leibnitz's rule for successive differentiation. 2

2. Answer any one question : 10 x 1

- (a) (i) If $y = \sin(m \sin^{-1}x)$, then prove that $(1-x^2)y_{n+2} - (2n+1)y_{n+1}x - (n^2 - m^2)y_n = 0$ and hence prove that $y_n(0) = 0$, for even n .

(ii) Find the value of p and q such that

$$\lim_{x \rightarrow 0} \frac{x(1 - p \cos x) + q \sin x}{x^3} = \frac{1}{3} \quad 10$$

- (b) (i) Trace the curve $r^2 = a^2 \cos 2\theta$. 5
- (ii) Find the envelope of circles whose centres lie on the rectangular hyperbola

$xy = c^2$ and which passes through its centre. 5

UNIT - II
(Calculus - II)

3. Answer any two questions : 2 x 2

(a) Show that the area of the circle $r = 2a \sin \theta$ is πa^2 . 2

(b) If $I_n = \int_0^{\pi/2} x^n \sin x dx$, n being positive integer > 1 , then show that

$$I_n + n(n-1)I_{n-2} = n \cdot \left(\frac{\pi}{2}\right)^{n-1} \quad 1 + 1$$

(c) Find the length of the circumference of a circle of radius a . 2

4. Answer any two questions : 5 x 2

(a) Find the volume of the solid generated by revolving the cardioid $r = a(1 - \cos \theta)$ about the initial line.

(4)

(b) If $I_n = \int_0^{\pi/2} \cos^{n-1} x \sin nx \, dx$, show that

$$2(n-1)I_n = 1 + (n-2)I_{n-1}$$

(c) Show that the area bounded by the parabolas

$$y^2 = 4ax \text{ and } x^2 = 4ay \text{ is } \frac{16}{3}a^2.$$

UNIT - III

(Geometry)

5. Answer any *three* questions : 3 × 2

(a) Find the equation of the right circular cylinder whose axis is z-axis and radius equals to 1.

(b) Find the values of c for which the plane $x + y + z = c$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$$

(c) Find the polar equation of the straight line passing through the points $(1, \frac{\pi}{2})$ and $(2, \pi)$.

(5)

(d) Find the nature of the quadric surface given by the equation

$$2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z = 5$$

(e) Under what condition the surface $yz + zx + xy = a^2$ may produce a parabola as a plane section by the plane $lx + my + nz = p$?

6. Answer any *one* question : 5 × 1

(a) If a sphere touches the planes $2x + 3y - 6z + 14 = 0$ and $2x + 3y - 6z + 42 = 0$ and if its centre lies on the straight line $2x + z = 0$, $y = 0$, find the equation of the sphere.

(b) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes at A, B, C . Find the equation of the cone generated by the straight lines drawn from the centre O to meet the circle ABC .

(6)

7. Answer any one question : 10×1

(a) (i) Show that the straight line $r \cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e \cos \theta$, if

$$(l \cos \alpha - ep)^2 = p^2 - l^2 \sin^2 \alpha.$$

(ii) Find the equations of the generating lines of the hyperboloid $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$, which passes through the point $(2, 3, -4)$.

(b) (i) A sphere of constant radius r passes through the origin and cuts the axes at A, B, C . Prove that the locus of the foot of the perpendicular from origin to the plane ABC is given by

$$(x^2 + y^2 + z^2)^2 (x^{-2} + y^{-2} + z^{-2}) = 4r^2.$$

(ii) Find the equation of the cylinder whose generator are parallel to the straight line $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 9, z = 1$.

(7)

UNIT - IV

(Differential Equation)

8. Answer any two questions : 2×2

(a) By which condition the ODE

$$M(x, y) dx + N(x, y) dy = 0 \text{ will be exact ?}$$

Is this condition necessary ?

(b) Determine the integrating factor of

$$(x^4 y^2 - y) dx + (x^2 y^4 - x) dy = 0$$

(c) The bacteria in a certain culture increase according to $dN/dt = 0.25 N$. If originally $N = 200$, find N when $t = 8$.

9. Answer any one question : 5×1

(a) Solve

$$(x^2 y^2 + xy + 1) y dx - (x^2 y^2 - xy + 1) x dy = 0$$