# Tower of Hanoi

developed by

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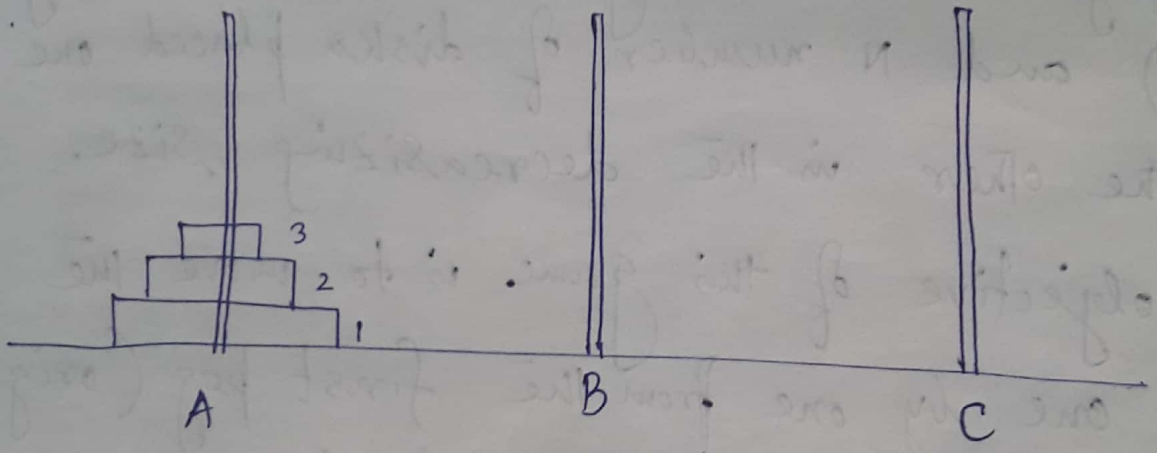
Tower of Hanoi is a very famous mathematical puzzle/game. In this game there are 3 pegs (tower) and N number of disks placed one over the other in the decreasing size. The objective of this game is to move the disks one by one from the first peg (origin) to the last peg (destination) with the help of auxiliary peg. There are some conditions in this Tower of Hanoi problem.

(i) only one disk can be moved among the towers at any given time.
(ii) Only the 'top' disk can be removed.
(iii) No large disk can be sit over a small disk.

Tower of Hanoi problem with N disks can be solved in $(2^N - 1)$ steps.

| N (disks) | No. of movement |
|---|---|
| 1 | $2^1 - 1 = 2 - 1 = 1$ |
| 2 | $2^2 - 1 = 4 - 1 = 3$ |
| 3 | $2^3 - 1 = 8 - 1 = 7$ |




For N=3 (No. of disks = 3)

Movements are given below:-

[A → C, A → B, C → B] Movement of top (N-1) disks from source peg (A) to auxiliary peg (B). That is movement of top 2 disks from peg A to peg B.

[A → C] Move $N^{th}$ disk, a larger disk from source peg (A) to destination peg (C).

[B→A, B→C, A→C] Move (N-1) disks from auxiliary peg (B) to destination peg (C). That is movement of 2 disks from peg B to peg C.

To solve this problem we will follow 3 simple steps recursively.

We will use a general notation :

TOWER (N, Beg, Aux, End)

TOWER denotes our procedure or algorithm.
N denotes the number of disks.
Beg is the initial (source) peg.
Aux is the auxiliary peg.
End is the final (destination) peg.

## Algorithm

/* N = Number of disks
/* Beg, Aux, End are the pegs */

TOWER (N, Beg, Aux, End)
Beg in if N = 1 then
print: Beg → End;

else
    Call TOWER(N-1, Beg, End, Aux);
    Call TOWER(1, Beg, Aux, End);
    Call TOWER(N-1, Aux, Beg, End);
end if
End

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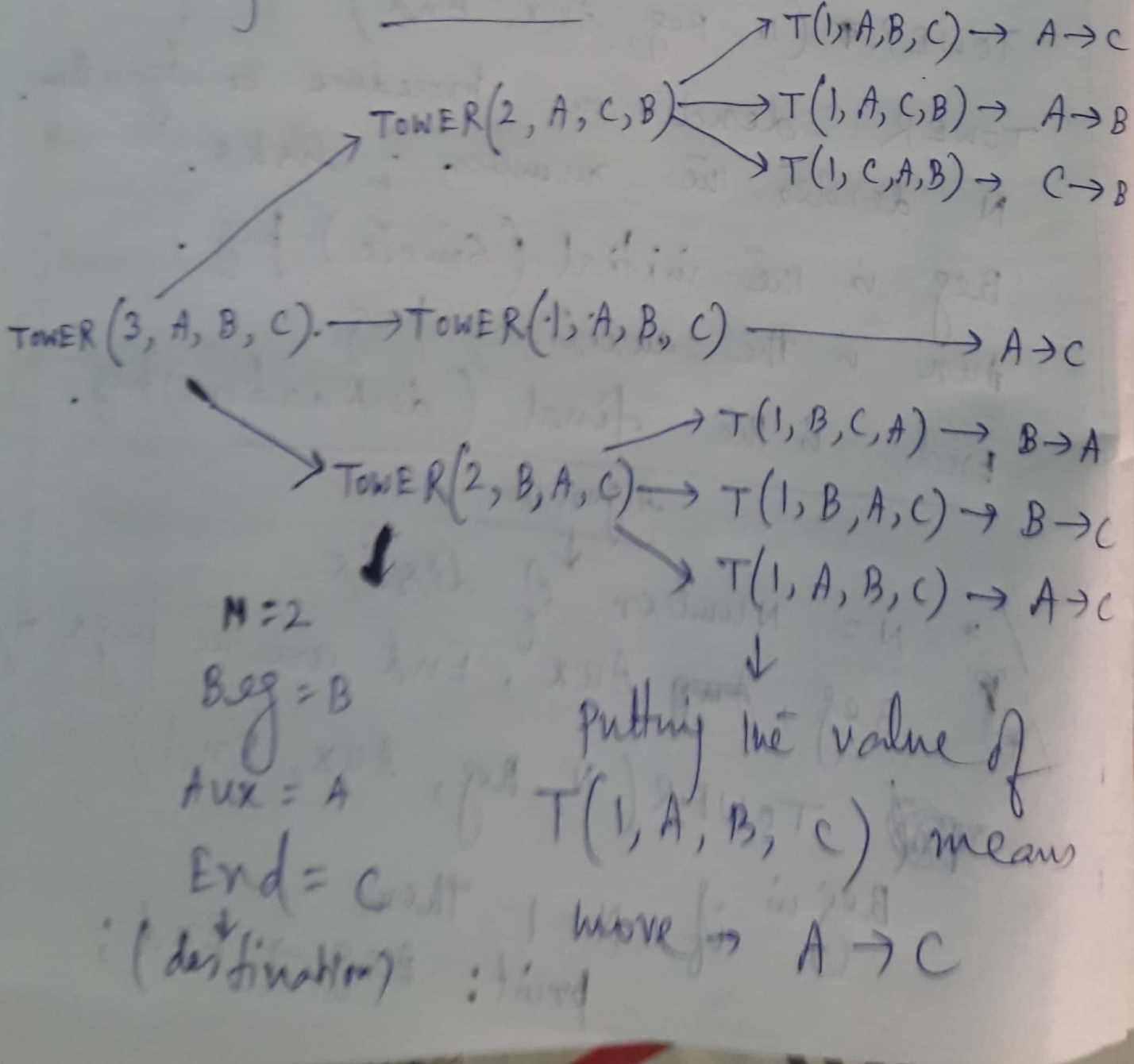
Illustration of Tower of Hanoi problem using N = 3 disks. (T → TOWER)



# Time Complexity of Tower of Hanoi

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$T(n) = C + 2T(n-1)$ / $1 + 2T(n-1)$

ek,  $T(n) = 1 + 2\left[1 + 2T(n-2)\right]$

$= 1 + 2.1 + 2.2\ T(n-2)$

$= 1 + 2 + 4\ T(n-2)$

$= 1 + 2 + 4 + 8 + \text{----} + 2^{n-1} + 2^{n}\ T(n-n)$

$= 1 + 2 + 4 + 8 + \text{----} + 2^{n-1} + 2^{n}.0$

$= 1 + 2 + 4 + 8 + \text{---} + 2^{n-1}$

(Since $T(0) = 0$ movement of no disk $= 0$)

$= \frac{2^n - 1}{(2-1)}$  [G.P Series]

$= 2^n - 1$

$\therefore T(n) = O(2^n)$