3. (a) In one dimensional Ising model, why is a closed loop of infinite radius considered in place of open chain? (1)

(b) Find out the expression for magnetization in an external magnetic field of B in case of this model. (6)

(c) Explain qualitatively that Ising model on square lattice can have spontaneous magnetization at non-zero temperature (3)

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(Internal Assessment – 10)

2017

M.Sc.

4th Semester Examination

PHYSICS

PAPER – PGS-401 (Gr. – A + B)

Full Marks : 50

Time : 2 Hours

The figures in the right hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Answer Q1 and any one from Q2 and Q3 for each of Groups A & B.

(Gr. A - *Relativity*)

1. Answer any five bits: 5 X 2 = 10

(i) Define tangent vectors and one-forms.

(ii) State the equivalence principle.

(iii) What do you mean "parallel-transport"?

(iv) Write down the Einstein field equation and explain it.

(v) Show that ${R_{abd}}^{c} + {R_{bda}}^{c} + {R_{dab}}^{c} = 0$ where ${R_{abd}}^{c}$ is Riemann curvature tensor.

(vi) What is "Chandrasekhar limit"?

(vii) Show that $ds^2 = g_{ab}dx^a dx^b$ is an invariant. Hence show that g_{ab} transform as covariant tensor.

(viii) A curve is specified by the following coordinates in terms of parameters 't':

$x^0 = \sqrt{3}ct;\ x^1 = ct;\ x^2 = ct_0 \cos\left(\frac{t}{t_0}\right);\ x^3 = ct_0 \sin\left(\frac{t}{t_0}\right)$. Determine whether the tangent vector at a typical point is space-like, time-like or null, where the space-time is Minkowskian.

(Turn Over)

2. (a) Write down Bianchi identity and prove this. (3)

(b) Suppose $f = \tilde{w} o v$ where f is a scalar and $\tilde{w}$ is one-form and v is tangent vector then show that $£_u\tilde{w} = \nabla_u\tilde{w} + \tilde{w} o \nabla_u$ or in component notation

$(£_u\tilde{w})_b v^b = \{u^a(\nabla_a w_b) + w_a(\nabla_b u^a)\}v^b.$ (3)

(c) Verify Newton's law from geodesic equation. (4)

3. (a) Discuss Gravitational red shift in Schwarzschild metric. (4)

(b) Write down the Kruskal-Sekeres extension of Schwarzschild metric and explain it. (4)

(c) Show that Schwarzschild singularity occurs at $r_s = \frac{2GM}{c^2}$. (2)

(Gr. B – *Statistical Mechanics-II*)

1. Answer any five bits: 5 X 2 = 10

(i) At low temperature specific heat of a system of Fermi gas is proportional to the absolute temperature (T) whereas that of a lattice is proportional to the cube of the absolute temperature - explain it.

(ii) Think about a system of two particles each having 3 energy states with energies 0, E and 2E. The temperature of the system being T. Write down the partition function of the system if the particles obey (a) Fermi-Dirac statistics, (b) Bose-Einstein statistics.

(iii) What is Bose condensation? What are basic differences between Bose condensation and vapour condensation?

(iv) A classical particle of mass m is in one dimensional motion lying between x=0 and x=L. The energy of the particle is between E and E+ΔE. What is the area of the region of phase space accessible to the particle?

…Continued

(v) A canonical system has two distinct energy levels +Δ and - Δ. What is the average energy?

(vi) What is the chemical potential for a gas of photons in thermal equilibrium? Give reason.

(vii) How does the susceptibility in Pauli's theory of paramagnetism differ from Langevin's theory?

(viii) From Clausius-Clapeyron equation establish the expression of the pressure of a vapour which is in equilibrium with a liquid/solid at absolute temperature T.

2. Consider a system of non-interacting bosons in thermal equilibrium at a temperature T. The equation of state is given by $\frac{P}{KT} = \frac{1}{\lambda^3} g_{5/2}(z)$ and $\frac{N-N_0}{V} = \frac{1}{\lambda^3} g_{3/2}(z)$ where Bose-Einstein integral $g_\nu(z) = \frac{1}{\Gamma(\nu)}\int_0^\infty dx \, \frac{x^{\nu-1}}{z^{-1}e^x - 1}$, $z = e^{\mu/KT}$. [The symbols have their usual meanings]

(a) Find the temperature at which Bose condensation occurs. (2)

(b) Find the fraction of particles present in ground state and in excited states. Draw a diagram indicating different phases of the system. (4)

(c) Calculate the entropy of the system both below and above the condensation temperature. State the physical significance of the result. (4)

(Turn Over)