2018

M.Sc.

2nd Semester Examination

PHYSICS

PAPER – PGS-201 (Gr. – A + B)

Full Marks : 50

Time : 2 Hours

The figures in the right hand margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Answer Q1 and any one from Q2 and Q3 for each of Groups A & B.

(Gr. A – *Quantum Mechanics-II*)

1. Answer any five bits: 5 X 2 = 10

(i) How does the eigenket of position operator transform under parity?

(ii) Show that $\gamma^{\lambda}\sigma^{\mu\nu}\gamma_{\lambda} = 0 \; where \; \sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]$ (Symbols have their usual meaning).

(iii) Write down the limitations of Klein-Gordon equation for explaining the relativistic quantum mechanics.

(iv) Find the value of $[\hat{J}^2, \hat{J}_+]$ where $\hat{J}$ is angular momentum operator.

(v) Prove that the Trace of odd number of Gamma (γ) matrices is zero.

(vi) Using perturbation theory show that the ground state of Hydrogen atom will not show the first order Stark effect.

(vii) Use the WKB method to estimate the energy levels of a one-dimensional harmonic oscillator.

(Turn Over)

(viii) Using the variational principle estimate the ground state energy of the delta function potential $V(x) = -\alpha\,\delta(x)$. Use the trial wave function $\psi(x) = \left(\frac{2b}{\pi}\right)^{1/4} e^{-bx^2}$, where b is a variational parameter. The expectation value of the kinetic energy $\langle T\rangle = \frac{h^2}{8m\pi^2} b$.

2. (a) Two identical bosons are placed in an infinite square well. They interact weakly with one another via the potential $V(x_1, x_2) = -aV_0\,\delta(x_1 - x_2)$. Use the first order perturbation theory to estimate the effect of particle-particle interaction on the energies of the ground state and first excited state. (2+3)

(b) Calculate the transition probability for a Harmonic perturbation. (5)

3. (a) Write the Hamiltonian of a Dirac particle in the electromagnetic field. Hence find the velocity operator of a Dirac particle. Show that the Dirac particle always moves with the speed of light. (2+2+2)

(b) A particle is initially (t < 0) in the ground state of an infinite, one dimensional potential well with wall at x = 0 and x = a. If the wall at x = a is now suddenly moved (at t = 0) to x = 8a, calculate the probability of finding the particle the ground state and the first excited state of the new potential well. (2+2)

(Gr. B – *Methods of Mathematical Physics-II*)

1. Answer any five bits: 5 X 2 = 10

(i) Prove that the Gaussian function $f(x) = e^{-x^2/2}$ is its own Fourier transform.

(ii) Find the Laplace transform of the Bessel function $J_0(x)$.

(iii) Solve: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 9z$.

(iv) State Dirichlet and Neumann boundary value problems.

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(v) Determine the elements of the finite cyclic group generated by the matrix $\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$ explicitly.

(vi) Prove that a group $(G, .)$ is commutative if $(a.b)^n = a^n . b^n$, for any three consecutive integers n and for all $a, b \in G$.

(vii) Let $(G, 0)$ be an abelian group. Prove that the nth power map $f : G \to G$ defined by $f(x) = x^n$ is a homomorphism from G to itself.

(viii) Define regular representation of a finite group and obtain the regular representation of the cyclic group $(C_3\,, \,.)$, where $C_3 = \{1, \omega, \omega^2\}$.

2. (a) Use Laplace transform to solve the partial differential equation:

$$\frac{\partial y(x,t)}{\partial t} = \frac{\partial^2 y(x,t)}{\partial x^2},$$

where $y(0,t) = 0$, $y(5,t) = 0$ and $y(x,0) = 10 \sin(4\pi x)$. (4)

(b) Prove that the set of all transformations of a variable x to x′ of the form x′ = ax + b, where a, b are real numbers and a ≠ 0, forms a Lie group. (3)

(c) If H is a subgroup of a commutative group $(G, .)$ then prove that the factor group $\left(\frac{G}{H}\,, *\right)$ is commutative. (3)

3. (a) Obtain the character table of Klein's 4-group V_4 and cyclic group C_4 of order 4. (4)

(b) Solve the integral equation: $F(t) = t^2 + \int_0^t \sin(t-u)\, F(u) du$. (3)

(c) Construct Green's function for the differential equation $\frac{\partial^2 y(x)}{\partial x^2} + y(x) = f(x)$ subject to the initial conditions $y(0) = 0$ and $y'(0) = 0$. (3)

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(Internal Assessment – 10)