2019

M.Sc.

2^{nd} Semester Examination

PHYSICS

COURSE – PHS 201 (Gr. – 201.1 & 201.2)

Full Marks : 50

Time : 2 Hours

The figures in the right hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Use separate answer scripts for Group 201.1 and Group 201.2.

Answer Q1, Q2 and any one from Q3 and Q4

(*Quantum Mechanics-II – PHS 201.1*)

1. Answer any two bits: **2 X 2 = 4**

(i) Suppose a spin $\frac{1}{2}$ particle is in the state $\chi = \frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 2 \end{pmatrix}$, what are the probabilities of getting $+\frac{\hbar}{2}$ and -$\frac{\hbar}{2}$, if we measure s_z?

(ii) Show that $\gamma^{\lambda}\sigma^{\mu\nu}\gamma^{\rho}\gamma_{\lambda} = 2\gamma^{\rho}\sigma^{\mu\nu} where\ \sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu},\gamma^{\nu}]$ (Symbols have their usual meaning).

(iii) Write down the limitations of Klein-Gordon equation for explaining the relativistic quantum mechanics.

(iv) Suppose we put a delta function bump in the centre of the infinite square well $H' = \alpha\delta\left(x - \frac{a}{2}\right)$, where α is a constant. Find the 1^{st} order correction to the allowed energies. Explain why the energies are not perturbed for even n.

(Turn Over)

2. Answer any two bits: **2 X 4 = 8**

(i) Calculate the Clebsch-Gordan coefficients for $j_1 = \frac{1}{2}$ and $j_2 = \frac{1}{2}$.

(ii) Using the WKB approximation to find the allowed energy (E_n) of an infinite square well with a shelf of height V_0 extending half way across where

$$V(x) = \begin{matrix} V_0 & for\ 0 < x < a/2 \\ 0 & for\ a/2 < x < a \\ \infty & otherwise \end{matrix}$$

(iii) Show that total angular momentum is a constant of motion for Dirac particle in central force field.

(iv) Show that $|1,0>$ is the eigenstate of S^2 where $\vec{S} = \vec{S}^{(1)} + \vec{S}^{(2)}$.

3. (i) The unperturbed wave function for the infinite square well are $\Psi_n^{(0)} = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$. Suppose we perturb the system by simply raising the floor of the well by a constant amount V_0. Find the 1st order correction of energy. (2)

(ii) Using variational principle calculate the ground state energy of delta potential, $V(x) = -\alpha\delta(x)$. (3)

(iii) Show that $Tr(\gamma^5\gamma^\mu\gamma^\nu) = 0$ (Symbols have their usual meaning). (3)

4. (i) Discuss the Dirac's hole theory. (3)

(ii) Write the covariant form of Dirac equation in coordinate representation. Hence find the free particle Dirac Hamiltonian. Also find the hermicity properties of γ^μ matrices. (1+1+2)

(iii) What do you mean "Zitterbewegung"? (1)

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(*Methods of Mathematical Physics-II - PHS 201.2*)

1. Answer any two bits: **2 X 2 = 4**

(i) Solve the partial differential equation $\frac{\partial^2 z}{\partial y^2} - z = 0$; given that when $y = 0, z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$.

(ii) Solve the integral equation $f(t) = \sin t - 2\int_0^t f(u)\cos(t-u)du$.

(iii) What is the order of nth dihedral group D_n? Prove that the fourth dihedral group D_4 has no subgroup of order 3.

(iv) Define Lie group with an example.

2. Answer any two bits: **2 X 4 = 8**

(i) Solve: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial z}{\partial x} = \sin(3x+4y) - e^{2x+y}$.

(ii) Find the Fourier transform of the function $f(x) = \begin{matrix} 1, if\ |x| \le a \\ 0, if\ |x| > a \end{matrix}$ and use it to evaluate $\int_0^\infty \frac{\sin^2 \omega a}{\omega^2} d\omega$.

(iii) Define class of a group. Prove that any two classes of a group are either identical or disjoint.

(iv) Show that the quotient group GL_2R / SL_2R is isomorphic with the multiplicative group R^*.

3. Find the Green's function for the boundary value problem $-\frac{d^2y(x)}{dx^2} = f(x)$ with boundary conditions $y(0) = 0, y(1) = 0$ and use it to solve the differential equation if $f(x) = \sin\pi x$. Verify the result by using Laplace transform. (8)

4. (a) Find the irreducible representation of the special unitary group SU(2). (5)
(b) Determine the character table for the group D_3. (3)

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(Internal Assessment – 10)