2. (a) Using $F_1 = \frac{1}{2}m\omega q^2 cotQ$ as generating function, obtain an expression for the displacement of LHO. (4)

(b) Two identical simple pendulums each having length l and a bob of mass m are coupled to each other by a horizontal massless spring constant k, the spring is not in stretched condition when the two bobs are in equilibrium. Find out the normal frequencies. (6)

3. (a) Prove that Poisson Bracket of two variable F and q remains invariant under canonical transformation. (3)

(b) Write down the Lagrangian of a particle moving under an inverse square attractive force field. Hence obtain the Hamiltonian of the particle. Find the cyclic coordinatein Lagrangian and Hamiltonian.

(3)

(c) Write down the Hamiltonian of a heavy symmetrical top. Comment on the precession related to its motion. (2+2)

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(Internal Assessment – 10)

2016

M.Sc.

2nd Semester Examination

PHYSICS

PAPER – PGS-201 (Gr. – A + B)

Full Marks : 50

Time : 2 Hours

The figures in the right hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Answer Q1 and any one from Q2 and Q3 for each of Groups A & B.

(Gr. A - *Methods of Mathematical Physics*)

1. Answer any five bits: 5 X 2 = 10

(i) Prove that the set of vectors $\{(1,2,2), (2,-2,1), (2,1,-2)\}$ is an orthogonal basis of the Euclidean space R^3 with standard inner product.

(ii) Let $(V, \langle\ ,\ \rangle)$ be an inner product space and x, $y \in V$. Show that $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$.

(iii) Examine the following matrix:

$$\begin{matrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{matrix}$$

(iv) Evaluate the value of $\Gamma\left(\frac{1}{2}\right)$ and use it to prove that $\Gamma\left(-\frac{3}{2}\right) = \frac{4\sqrt{\pi}}{3}$.

(v) If $erf(x)$ be the error function of x, show that

$$\int_0^\infty e^{-x^2-2bx}\,dx = \frac{\sqrt{\pi}}{2}e^{b^2}[1-erf(x)].$$

(Turn Over)

(vi) Using $f(r\,e^{i\theta}) = R(r,\ \theta)e^{i\Theta(r,\ \theta)}$ in which $R(r,\ \theta)$ and $\Theta(r,\ \theta)$ are differentiable real functions of r and θ, show that the Cauchy-Riemann conditions in polar coordinates become

$$\frac{\partial R}{\partial r} = \frac{R}{r}\frac{\partial \Theta}{\partial r} \text{ and } \frac{1}{r}\frac{\partial R}{\partial r} = -R\frac{\partial \Theta}{\partial r}.$$

(vii) Find out the residues of the function $f(z) = \frac{z^3}{(z-1)\,(z-2)(z-3)}$ at infinity and z = 3.

(viiii) Discuss the nature of singularity of the function $f(z) = e^{1/z}$ at z = 0.

2. (a) Two matrices $\boldsymbol{U}$ and H are related by $U = e^{i\alpha H}$, α is real. If H is Hermitian, Show that U is unitary. (4)

(b) For $x > 0$, Prove that $J_{\frac{-3}{2}}(x) = \left(-sinx - \frac{cosx}{x}\right)\sqrt{\frac{2}{\pi x}}$. (3)

(c) Evaluate $\oint_C \frac{dz}{z^2-1}$ where C is the circle $|z| = 2$. (3)

3. (a) Solve the hypergeometric equation:

$x(1-x)\frac{d^2y}{dx^2} + [\gamma - (1+\alpha+\beta)x]\frac{dy}{dx} - x\beta y = 0$ in series near $x = 0$. (4)

(b) State and prove Cauchy's residue theorem. (3)

(c) Apply the Gram-Schmidt process to obtain an orthogonal basis from the basis set $\{1,\ x,\ x^2\}$ of the inner product space

$P_2(R)$ = {p(x): p(x) is a polynomial of degree ≤ 2 with real coefficients}

with the inner product $\langle f(x), g(x)\rangle = \int_{-1}^{1} f(t)g(t)dt.$ (3)

…Continued

(Gr. B – *Classical Mechanics*)

1. Answer any five bits: 5 X 2 = 10

(i) Consider a system of N particles subjected to k independent holonomic constraints that do not depend on time explicitly. Then transformation from Cartesian $3N$ coordinates to f-independent generalized coordinates q_i have the following form:

$x_j = x_j\,(q_1, q_2, \ldots, q_f)$, $j = 1, 2, 3, \ldots, 3N$ and $f = 3N - k$. Show that the kinetic energy can be written as a quadratic form $\frac{1}{2}\sum_{ij} a_{ij}\dot{q}_i\,\dot{q}_j.$

(ii) The Lagrangian of a charged particle is given by $L = \frac{1}{2}\,mv^2 - q\varphi + q(\vec{v}.\vec{A})$. Find its Hamiltonian

(iii) Prove that the generating function $F = \sum_i q_i p_i$ generates the identity transformation.

(iv) Show that the transformation with $P = \frac{1}{2}(p^2+q^2)$ and $Q = tan^{-1}\left(\frac{q}{p}\right)$ is canonical.

(v) Distinguish between stable equilibrium and unstable equilibrium with suitable examples.

(vi) Show that for a rigid body torque is due to the external force only, internal force does not contribute at all.

(vii) Show that in the body system, the axes of the distance of the tangent plane from the origin remain invariant.

(viii) Consider any arbitrary function F of coordinates (q_k), canonical momenta (p_k) and time (t). Show that $\frac{dF}{dt} = \frac{\partial F}{\partial t} + [F,H]_{PB}$.

(Turn Over)