(d) The primitive translation vectors of two dimensional lattice are $\vec{a} = 2\hat{\imath} + \hat{\jmath}$, $\vec{b} = 2\hat{\jmath}$ and $\vec{c} = \hat{k}$. Determine the primitive translation vectors in its reciprocal space.

2. Answer any two bits: 2X4 = 8

(a) Bragg's angles are observed at 12.3°, 14.1°, 20.2° and 24.0° of X-ray powder diffraction photograph of a cubic structure recorded for X-ray wavelength 1.54A. Assign Miller indices to these lines and find the lattice constant.

(b) Prove that effective number of free electrons in a sold is maximum when the outermost band is half filled. Explain why some metal have positive Hall coefficient?

(c) Discuss how the anharmonic vibration leads to the thermal expansion of solids

(d)What is Van Hove singularity?

3. (a) Show the variation of energy, velocity and effective mass of an electron in a solid. What is crystal momentum?
(b) A two-dimensional material has square lattice with lattice constant a= 0.3 nm. The dispersion relations for electro energies in the conduction and valance bands given by $\epsilon_c(k) = 6 - 2(cosk_x a + cosk_y a)$ and $\epsilon_v(k) = -2 + (cosk_x a + cosk_y a)$. (i) Indicate the value and position of the band gap. (ii) Determine the effective masses of both electrons and holes. (3+2+3)

4. (a) Calculate the density of states for the first zone of a SC lattice according to the empty lattice model. (b) Plot g(E), and determine the energy at which g(E) has its maximum. (c) Explain qualitatively the behaviour of this curve. (4+2+2)

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Internal Assessment-10

2019

M.Sc.

1st Semester Examination

PHYSICS

PAPER – PHS-(102.1+ 102.2)

Full Marks : 50

Time : 2 Hours

Use separate answer scripts for Group 102.1 and Group 102.2

(Quantum Mechanics I – PHS 102.1)

Answer Q1, Q2 and any one from Q3 and Q4

1. Answer any two bits: 2x2 = 4

(a) Show that the eigenvalues of an anti-hermitian operator are purely imaginary.

(b) Given $a^{\dagger}a|n\rangle = n|n\rangle$ and $[a, a^{\dagger}] = 1$, show that $a|n\rangle = \sqrt{n}\,|n-1\rangle$ for the one-dimensional oscillator.

(c) Consider a virus of size 10A. Suppose that its density is equal to that of water (in g/cm^2) and the virus is located to the space that is approximately equal to its size. Compute the minimum speed of the virus.

(d) A particle of mass 'm' is constrained to move in a truncated one dimensional harmonic potential well ($x > 0$). Let the wave function of the particle be given by $\psi(x) = -\frac{1}{\sqrt{5}}\psi_0 + \frac{2}{\sqrt{5}}\psi_1$ where ψ_0 and ψ_1 are the eigen functions of the ground state and the first excited state respectively. Find out the energy expectation value.

2. Answer any two bits: 2x4 = 8

(Turn Over)

(a) In a region of space a particle of mass m with *zero energy* has a time-independent wave function $\psi(x) = Ax\, exp^{\left(-\frac{x^2}{L^2}\right)}$, where A and L are constants. Compute the potential energy $V(x)$ and sketch it as a function of x. (1+3)

(b) (i) What is the physical meaning of the use of the boundary conditions of the wave function and its first derivative in the quantum mechanical potential problem?

(ii) A particle is confined to a box of length L in one dimension. It is initially in the ground state. Suddenly, one wall of the box is moved outward making a new box of length $3L$. What is the probability that the particle is in the ground state of the new box?. (1+3)

(c) Consider a one-dimensional problem. Let the translation operator $T(a)$ describe the operation $T(a)\psi(x) = \psi(x + a)$, where a is a constant displacement. You can assume $T(a) = \exp[i.a.p/\hbar]$ where p is the momentum operator

.(i) Show that this operator commutes with the Hamiltonian, $H = -(\hbar^2/2m)\partial^2/\partial x^2 + U(x)$, if the potential has the periodic property $U(x) = U(x + a)$.

(ii) Let $\psi(x)$ be an eigenstate of $T(a)$ with eigenvalue c. Show that c is a constant of motion. (2+2)

(d) (i) Show that the energy levels of particle which is free except that it is constrained to move on the surface of '*a'* sphere of radius 'a' do not depend on the azimuthal quantum number. Note that for such rigid rotator the Hamiltonian can be expressed as $H = \frac{\hat{L}^2}{2ma^2}$.

(ii) Estimate the degeneracy of the bound states of Hydrogen atom. (2+2)

3.(i) Consider a spinless particle of mass m moving in a 3D potential:

(Continued)

$$V(x, y, z) = \frac{1}{2} m\omega^2 z^2, 0 < x < a, 0 < y < a$$
$$= \infty, \text{elsewhere.}$$

(a) Write down the energy and the wave function of this particle.

(b) Assuming that $\omega > 3\pi^2\hbar/2ma^2$, find the energies corresponding to the ground state and the first excited state. (2+2)

(ii) What is Interaction picture? Find the Schrodinger equation in the Interaction picture. (1+3)

4.(i) Show that the transformation matrix which changes from one basis to another is unitary. (2)

(ii) The normalized wave functions of a Hydrogen atom are denoted by $\psi_{n,l,m}$, where the subscripts carry their usual meanings. Now consider an electron in the mixed state $\psi(x) = \frac{1}{\sqrt{5}}\psi_{1,0,0}(x) + \frac{2}{\sqrt{5}}\psi_{2,1,0}(x)$. Find the expectation value of energy of the electron in eV. (3)

(iii) If $Y = \frac{1}{\sqrt{2}}[Y_{2,2} - Y_{2,-2}]$, where $Y_{l,m}$ are spherical harmonics, check whether Y is an eigen function of both L^2 and L_z or not? (3)

(*Solid State Physics I – PHS 102B*)

Answer Q1, Q2 and any one from Q3 and Q4

1. Answer any two bits: 2X2 = 4

(a) What is Miller-Bravis indices? Explain its necessity.

(b) Explain why optical branch does not arise in monoatomic lattice.

(c) How will you experimentally distinguish between single and poly crystalline material?

(Turn Over)