2019

M.Sc.

1st Semester Examination

PHYSICS

PAPER – PHS-101 (Gr. – 101.1+101.2)

Full Marks : 50

Time : 2 Hours

Use separate answer scripts for 101.1 and 101.2

(Methods of Mathematical Physics I - PHS 101.1)

Answer Q1, Q2 and any one from Q3 and Q4

1. Answer any two bits: 2x2 = 4

(a) Find the eigen values of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and justify the eigen values.

(b) Find the Laurent series expansion of the function $f(z) = \frac{1}{(z-a)(z-b)}, a \neq b$ about z = 0 in the region $0 < |a| < |z| < |b|$.

(c) Find the value of $\Gamma\left(\frac{21}{2}\right)\Gamma\left(-\frac{5}{2}\right)$.

(d) Prove that $\int_0^\infty e^{-x^2-2bx}\,dx = \frac{\sqrt{\pi}}{2}e^{b^2}.erf_c(b)$ where $erf_c(x)$ denotes the complementary error function of the error function $erf(x)$.

2. Answer any two bits: 2x4 = 8

(a) Evaluate : $\int_{|z|=1} \frac{e^{4z}-1}{coshz-2sinhz}dz$.

(b) State and prove Bessel's inequality in a Euclidean space .

(c) solve in series $x(1-x)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$ near $x = 0$.

(Turn Over)

(d) If m and n are positive integers then find the value of $\int_0^\infty L_m(x)\, L_n(x) dx$, where $L_n(x)$ represents the Laguerre polynomial of degree n.

3. (a) Prove that the function $f(z) = z\, Im\, z,\ z \in ¢$ is a differential equation at origin but it is not differentiable at any point of the punctured disc $\{\ z:\ z \in ¢$ and $0 < |z| < r$, where r is a positive real number.

(b) Prove that the function $f(z) = \frac{e^z - 1}{z(z-1)}$ has a removable singularity at $z = 0$ whereas it has a simple pole at $z = 2$. Prove also that it has an essential singularity at $z = \infty$.

4. (a) Apply Gram-Schmidt process to the subset *S = { (1, i, 2-i, -1), (2+3i, 3i, 1-i, 2i), (-1+7i, 6+10i, 11-4i, 3+4i)}* of the inner product space C^4 with standard inner product to obtain an orthogonal basis span (S).

(b) Find an orthogonal matrix P such that $P^{-1}A\,P$ is a diagonal matrix, where $A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$.

(*Classical Mechanics – PHS 101.2*)

Answer Q1, Q2 and any one from Q3 and Q4

1. Answer any two bits: 2x2 = 4

(a) a particle moves in a plane under the influence of a force acting towards the center given by $F = \frac{1}{r^2}\left(1 - \frac{\dot{r}^2 - 2r\,\ddot{r}}{c^2}\right)$, where r is the distance of the particle from the center of the force. Find the generalized potential that will result in such a force.

(b) Check the following transformation given below is canonical or not

(Continued)

$Q = log\left(\frac{1}{q}\sin p\right),\ P = q\cot p.$

(c) A particle is constrained to move on a plane curve $xy = constant$ under gravity. Obtain the Lagrange's equation.

(d) Consider a particle of mass m moving in one dimensional under a force with the potential $U(x) = k(2x^3 - 5x^2 + 4x)$ where the constant $k > 0$. Show that the point x =1 corresponds to a stable equilibrium position for the particle.

2. Answer any two bits: 2x4 = 8

(a) What is cyclic coordinate? Show that if a coordinate is cyclic in Lagrangian, it will also be cyclic in Hamiltonian. Explain with one example.

(b) Discuss how the isotropy of space in Lagrangian leads to conservation of momentum of a system of particles.

(c) If the Hamiltonian of system is $H = \frac{p^2}{2} - \frac{q^2}{2}$, show that $F = \frac{pq}{2} - Ht$ is a constant of motion.

(d) State and prove principle of least action.

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3. (a) Find the normal frequencies of a linear tri-atomic molecule. (b) What are normal coordinates? (c) Find the normal coordinates in this case. (5+1+2)

4. Find out the frequency of a linear harmonic oscillator using action-angle variable method. Starting from the time dependent Schrodinger equation obtain the Hamilton-Jacobi equation. (5+3)

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Internal Assessment-10