2017

M.Sc.

1st Semester Examination

PHYSICS

PAPER – PHS-101 (Gr. – A + B)

Full Marks : 50

Time : 2 Hours

(Methods of Mathematical Physics I - PHS 101A)

Answer Q1 and any one from Q2 and Q3

1. Answer any five bits: 5x2 = 10

(a) Prove that the function $f: \not\subset \to \not\subset$ defined by $f(z) = Im\, Z, Z \in \not\subset$ is nowhere analytic.

(b) If A and B are Hermitian matrices then show that $i(AB - BA)$ is Hermitian.

(c) Discuss the nature of singularity of the function $f(z) = \frac{e^z - 1}{z(z-2)}$ at $z = \infty$.

(d) Prove that the set vectors $\{(2,3,-1),(1,-2,-4),(2,-1,-1)\}$ is an orthogonal basis of the Euclidean space R^3 with standard inner product.

(e) Show that $\int_0^\infty e^{-x} L_n(x) L_m(x) dx = \delta_{mn}$.

(f) Prove that with $z = re^{i\theta}$, the Cauchy-Riemann conditions are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \qquad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

(Turn Over)

(g) Prove that the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$ is diagonizable.

(h) Find the value of $\Gamma(-\frac{3}{2})$.

2. (a) Prove that $\int_0^\infty \frac{\log x}{(1+x^2)^2} dx = -\frac{\pi}{4}$. (4)

(b) Establish Triangle Inequality in a complex inner product space. (3)

(c) Solve in series the differential equation:

$\frac{d^2u}{dx^2} + (x+2)\frac{du}{dx} + u = 1 + 4x + x^2$ as a power series in $(x+2)$. (3)

3. (a) Show that $x = 0$ is a regular singular point of the differential equation $(x^3 + 2x)\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6xy = 0$ and solve the equation in series about $x = 0$. (4)

(b) Prove that three eigen vectors corresponding to three distinct eigen values of a square matrix over a field F are linearly independent. (3)

(c) Find the Laurent series expansion of the function $f(z) = \frac{1}{(z-a)(z-b)}$ about $z = 0$ in the regions $(i) 0 < |a| < |z| < |b|$ *and*

$(ii) |z| > |b| > |a|$ (3)

(*Classical Mechanics – PHS 101B*)
Answer Q1 and any one from Q2 and Q3

1. Answer any five bits: 5X2 = 10

(a) Show that the transformation defined by $q = \sqrt{2P}\sin Q$, *and*

$p = \sqrt{2P}\cos Q$ is canonical using Poisson bracket condition.

(*Continued*)

(b) The Lagrangian of a system is described as $L = \dot{q}^2 - q\dot{q}$. Derive the Hamiltonian of the system.

(c)Derive Hamilton's canonical equation in terms of Poisson bracket.

(d) Solve: $\Delta\int_{t_1}^{t_2} \sum_k p_k\, q_k dt = \delta\int_{t_1}^{t_2} L\, dt + (L+H)[\Delta t]_{t1}^{t2}$.

(e) Prove the following relation between Lagrange and Poisson brackets: $\sum_{i=1}^{2n}\{u_i, u_j\}[u_i, u_k] = \delta_{jk}$

(f) A particle of mass m moves in a potential $v(x) = \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\mu v^2$ where x is the position co-ordinate, v is the velocity, ω and μ are the constants. Find the canonically conjugate momentum of the particle.

(g) Show that if the Lagrangian does not depend explicitly on time, then the energy is conserved.

(h) Prove that if $F(q,p,t)$ and $G(q,p,t)$ are two integrals of motion, then $[F,G]$ is also integrals of motion.

2. (a) Deduce Hamilton's principle from D'Alembert's principle. What is modified Hamilton's principle? (5+1)

(b) A particle of mass m moves in a two-dimensional potential $V(x,y) = -(k_1x^2 + k_2y^2)$. Write down (i) the Lagrangian, (ii) the Hamiltonian and (iii) the Lagrange's equations of motion. (1+1+2)

3. (a) Starting from the time-dependent Schrödinger equation obtain Hamilton-Jacobi equation. What is Hamilton's principal function? (3+1)

(b) Using Hamilton-Jacobi theory, solve the Kepler's problem for a particle of mass m in an inverse-square central force field. The Hamiltonian being given by $H = \frac{1}{2m}\left[p_r^2 + \frac{p_\theta^2}{r^2}\right] - \frac{K}{r}$. The symbols have their usual meanings. (6)

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Internal Assessment-10